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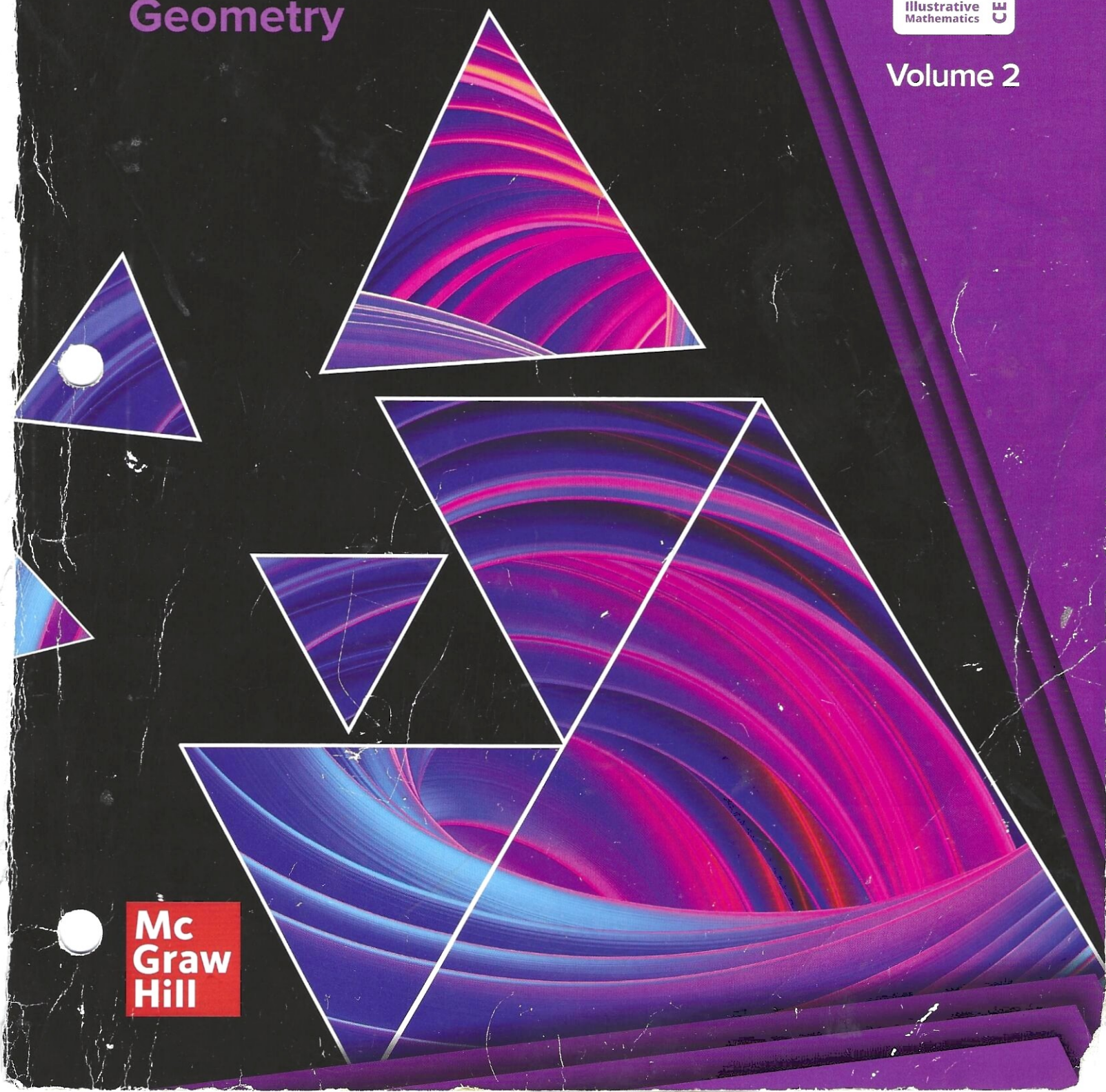

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Part 4 of 4–Unit 8 (Last chapter)

Volume 2

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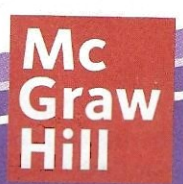
Geometry



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Illustrative Mathematics is designed to provide opportunities for you to learn math by doing math. By engaging in problem-solving activities, you will make and test your own conclusions and try out different strategies as you learn math.

Let's get started!

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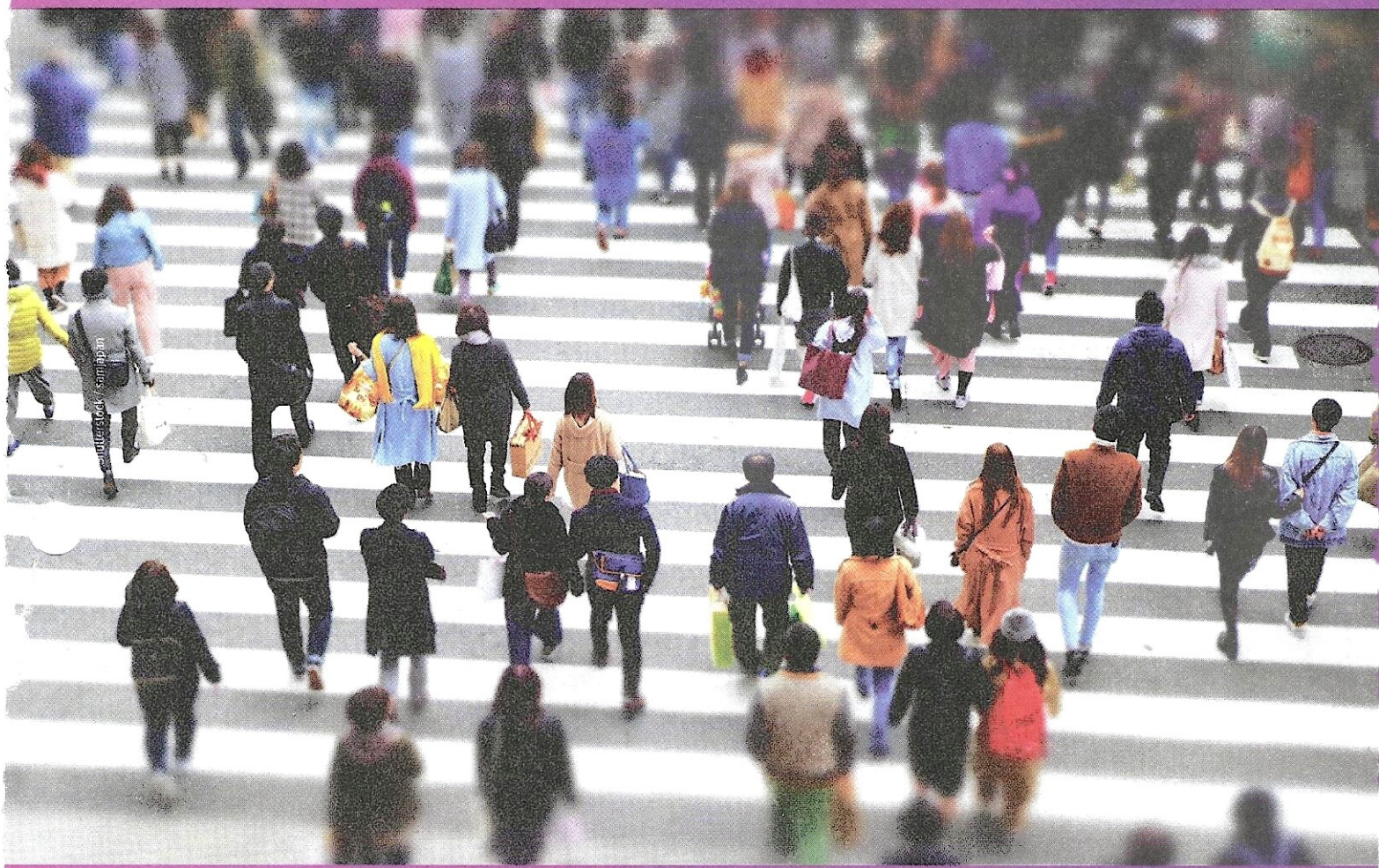
Unit 8

Conditional Probability



Knowing how to find the probability of an event allows you to make predictions about the population. You'll learn more about conditional probability in this unit.

Topics

- Up to Chance
- Combining Events
- Related Events
- Conditional Probability

Unit 8

Conditional Probability

Up to Chance

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Lesson 8-1

Up to Chance

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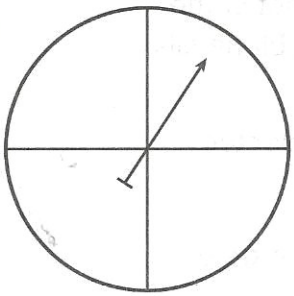
Learning Goal Let's explore chance.

Warm Up

1.1 Which One Doesn't Belong: Spinners

Which one doesn't belong?

A



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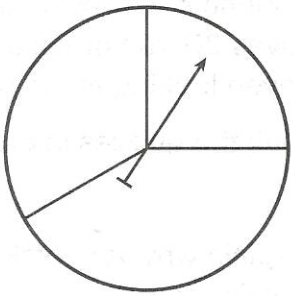

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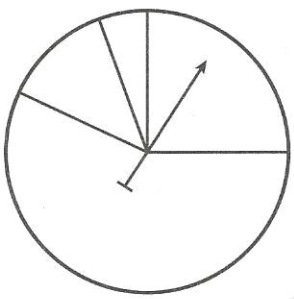

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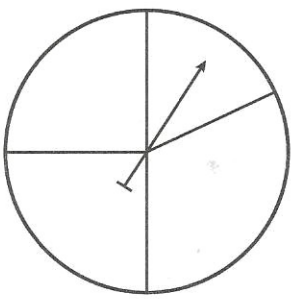


Activity

1.2 You're Saying There's a Chance?

In Elena's Spanish class, they have a quiz every two weeks.

- For the first quiz of the year, Elena takes time to study and understands the material very well. The quiz involves 20 multiple choice questions with possible answers A, B, C, or D. Elena tries her best to answer the questions correctly.
- For the second quiz of the year, Elena has been absent a lot and does not understand the material at all. The quiz involves 20 multiple choice questions with possible answers A, B, C, or D. Elena fills in the answer sheet without even looking at the questions.
- For the third quiz of the year, Elena is still lost in the class and has not come in for any help. The quiz involves 20 true or false questions. Again, Elena fills in the answer sheet without even looking at the questions.

1. Based on the description, rank the quizzes in order from worst expected grade to best.

2. For each of the 3 quizzes, explain why you think chance played a large or small role in determining Elena's score.

3. For the second and third quizzes, Elena did not look at the questions. Explain why you think she might do better on one than the other.

4. What percentage of the questions do you think Elena will get right on each quiz? Explain your reasoning.

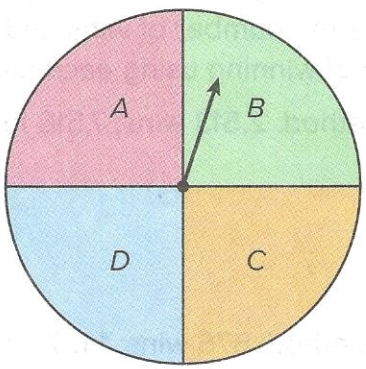
NAME _____ DATE _____ PERIOD _____

5. For the second quiz, the teacher made a mistake and each of the questions had two correct answers. The teacher accepted either of the correct answers for full credit. What percentage of the questions do you think Elena will get right on the second quiz with the new scoring? Explain your reasoning.

Activity

1.3 A Fair Game

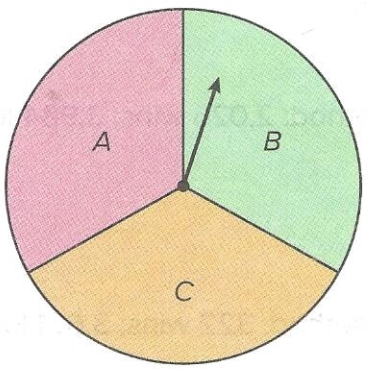
Han, Clare, Mai, and Kiran are inventing a game for the county fair. Players will spin a spinner and if it points to the section labeled B, then the player will win a prize.



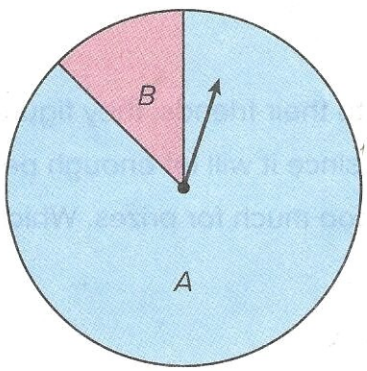
Han says, "I think this spinner is a good one. What do you think?"

Clare says, "I like the spinner, but I think people should be able to spin again if they don't win the first time."

Mai says, "What if we just make 3 sections like this?"



Kiran says, "I think it might make more sense if we just do two sections like this."



1. Put the proposals in order of the **probability** that a player will win using that method from least to greatest. Explain your reasoning. Share your explanation with a partner.

2. Each student writes a computer program to play the game using the method they suggested. The computer runs the program for a short time and reports the number of wins and losses. Use the results to estimate the probability of winning using each method.
   - Han's method: 2,513 wins; 7,516 losses.
   - Clare's method: 876 wins; 1,127 losses.
   - Mai's method: 2,026 wins; 3,984 losses.
   - Kiran's method: 322 wins; 3,621 losses.

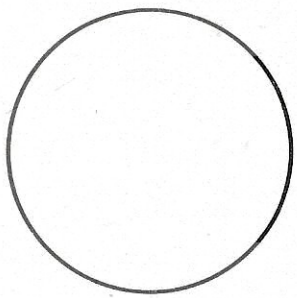
3. By talking to their friends, they figure out that a good probability for winning is about $\frac{1}{5}$ since it will let enough people win to draw in customers, but not cost them too much for prizes. Which method fits this best?

NAME _____ DATE _____ PERIOD _____

4. Before they settled on a spinner game, they considered other things at their booth. Which of these suggestions would be considered chance experiments?
   - A watermelon-eating contest. The fastest to eat a wedge of watermelon wins the prize.
   - Two cubes have one face labeled "Win!" If both cubes land with the "Win!" side facing up, the player wins a prize.
   - A ball is placed under one of five cups. The cups are shuffled around under a cover so the player cannot see how they are moved. The player chooses one of the cups and wins a prize if it has the ball under it.
   - Players push a button that starts lighting up different regions on a board. The game is rigged so that every fifth person wins.

Are you ready for more?

1. Draw a spinner with 3 or more sections that are not all the same size.



2. Your spinner is spun 1,000 times. Estimate the number of times you would expect the spinner to land on each section. Explain your reasoning.

3. A different spinner with only two sections is spun 1,000 times. It landed on one section 136 times and the other section 864 times. Draw a spinner that is likely to produce similar results if spun 1,000 times. Explain your reasoning.

Summary

Up to Chance

Few things are certain in life, but we can look more closely at uncertain processes, called chance experiments, to get a better idea of the likelihood of a particular thing happening. Lots of times, it is possible to recognize several different results that may happen. These results are called outcomes. The collection of all possible outcomes is called the sample space. An event is a group of outcomes from the sample space.

For example, flipping a coin is a chance experiment that has possible outcomes of landing with heads facing up or tails facing up. Sometimes, one outcome is more likely than another. For example, when walking outside in a thunderstorm, it is much more likely you will get hit by a raindrop than a lightning bolt.

To put a number to this likelihood is to find or estimate its **probability**. The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. It is estimated that the probability of getting hit by lightning in the United States in one year is about $\frac{1}{700{,}000}$, which means that, each year in the United States, a person has a 0.00014% ($1 \div 700{,}000 \approx 0.0000014$) chance of getting hit by lightning.

Glossary

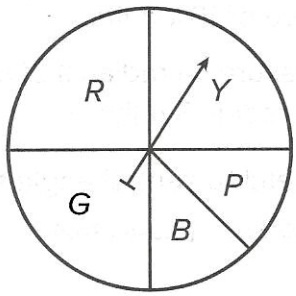
chance experiment
event
outcome
probability
sample space

NAME _____ DATE _____ PERIOD _____

Practice

Up to Chance

1. What is the probability of the spinner landing on the section labeled G?



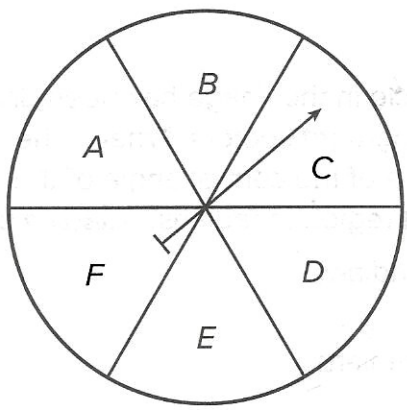
   (A) $\frac{1}{2}$

   (B) $\frac{1}{4}$

   (C) $\frac{1}{5}$

   (D) $\frac{1}{8}$

2. This spinner is spun 1,000 times. Which of these is the best estimate for the number of times it would be expected to land on the section labeled B?



   (A) 16

   (B) 60

   (C) 160

   (D) 600

3. On an assignment, there are two true or false questions. You have no idea what the correct answer is to either one so you guess.

   a. What is the probability that you get both of them right by guessing? Explain your answer.

   b. What is the probability that you get exactly one of them right by guessing? Explain your answer.

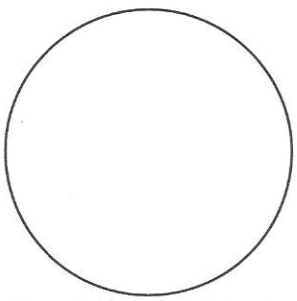
4. Respond to each question. (Lesson 7-13)

   a. Find the length of an arc with central angle $\frac{2\pi}{3}$ radians and a radius of 7 units.

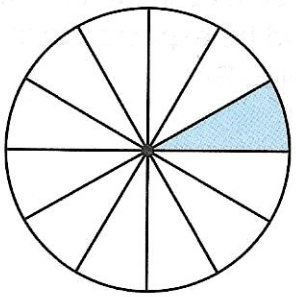
   b. Find the length of an arc with central angle $\frac{\pi}{4}$ radians and a circumference of 10π units.

   c. Find the measure (in radians) of a central angle of an arc with arc length 12 units and radius 3 units.

5. In the circle, sketch a central angle that measures $\frac{\pi}{2}$ radians. (Lesson 7-12)



6. The circle in the image has been divided into congruent sectors. What is the measure of the central angle of the shaded region in radians? (Lesson 7-12)



   (A.) $\frac{1}{12}$ radians

   (B.) $\frac{\pi}{12}$ radians

   (C.) $\frac{1}{6}$ radians

   (D.) $\frac{\pi}{6}$ radians

7. Select **all** true statements. (Lesson 7-6)

   (A.) The incenter of a triangle is the intersection of the angle bisectors.

   (B.) The incenter is the same distance from all the vertices of the triangle.

   (C.) The incenter is the same distance from all sides of the triangle.

   (D.) In order to construct the incenter, all 3 angle bisectors must be constructed.

   (E.) The incenter is the intersection of the perpendicular bisectors of each side.

Lesson 8-2

Playing with Probability

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's explore probability.

Warm Up

2.1 Taking Names

Your teacher will give your group a bag containing slips of paper with names on them. It is important not to open the bag to read the slips at any time. It is important to record your group's data in writing.

Every time a student in the class is notably helpful, a teacher puts their name on a slip of paper and puts it into a bag. If the same student is helpful more than once, their name can be entered multiple times. At the end of the month, the teacher draws several names for prizes. Follow these steps to collect data about the names in the bag:

1. Shake the bag, then draw out only 1 slip of paper.
2. Read the name you drew out loud so that everyone in the group can record the name.
3. Return the slip of paper to the bag and pass the bag to the next person in the group.
4. Repeat these steps until each person in the group has had a chance to draw at least 3 names.

Activity

2.2 Who Was Helpful?

Use the data your group collected in the warm-up to answer the questions.

1. Based on the data you collected, estimate the probability of drawing each of these names from your bag. Explain or show your reasoning.
   a. Clare

   b. Mai

   c. Priya

   d. Elena

   e. Jada

   f. Han

   g. Andre

   h. Diego

   i. Noah

2. There are 15 slips of paper in the bag. What names do you think are written on the slips? Explain your reasoning.

3. If you are allowed to keep going around the group, drawing names and replacing them until you had 100 names drawn, how do you think that affects your understanding of what is in the bag?

NAME _____ DATE _____ PERIOD _____

4. The next month, the bag contains 15 slips as well. Mai's name is included 5 times, Clare's name 4 times, Han's name 3 times, Diego's name 2 times, and Jada's name 1 time. The teacher draws names one at a time, replacing them each time. What might the teacher's list of names drawn look like if she draws 10 times? Is this the only list of names drawn that is possible? Explain your reasoning.

Activity

2.3 Probability Words

Take turns with your partner coming up with words that have the probabilities given when selecting a letter at random from the word. Each person should try to come up with one word for each situation.

1. $P(\text{vowel}) = \frac{1}{3}$. $P(\text{consonant}) = \frac{2}{3}$.

2. $P(\text{vowel}) = \frac{2}{3}$. $P(\text{consonant}) = \frac{1}{3}$.

3. $P(\text{vowel}) = 0.5$. $P(\text{T}) = \frac{1}{4}$.

4. $P(\text{S}) = 0.5$. $P(\text{vowel}) = 0.25$.

5. Think of a word and give your partner at least 2 clues about the word using probability of certain letters or types of letter.

Are you ready for more?

Each of the whole numbers from 1 to 25 is written on a slip of paper and placed in a bag.

1. Calculate each probability.

   a. P(prime)

   b. P(divisible by 3 but not 2)

   c. P(multiple of 5)

   d. P(greater than 20)

   e. P(multiple of 12 and less than 20)

2. Use this situation to create two of your own probability questions that give a probability of $\frac{1}{25}$ as the answer.

3. Use this situation to create two of your own probability questions that give a probability of $\frac{3}{25}$ as the answer.

NAME _____ DATE _____ PERIOD _____

Summary

Playing with Probability

Some probabilities are estimated by doing an experiment, or sometimes simulating the experiment many times and collecting data about how often outcomes come up. For example, a radio show holds a contest in which callers are entered for a chance to win a ticket to a concert in town. The probability of each caller winning is estimated by considering previous similar contests and comparing the number of callers to the number of ticket winners. If a previous contest had 327 callers and 5 ticket winners, then the probability of winning a ticket can be written:

$P(\text{winning a ticket}) = \frac{5}{327}$ or $P(\text{winning a ticket}) \approx 0.015$

Which means that each caller has about a 1.5% chance of winning a ticket to the concert.

Other probabilities can be determined by recognizing the expected relative likelihood of outcomes among all possible outcomes. For example, we know that the probability of rolling a 2 on a standard number cube is $\frac{1}{6}$ since there are 6 equally likely outcomes in the sample space for each roll and the event of rolling a 2 is one of those outcomes. This can be written as $P(\text{rolling a 2}) = \frac{1}{6}$.

Practice

Playing with Probability

1. Six papers are placed in a bag with names written on them. The names are: Lin, Mai, Mai, Noah, Priya, and Priya. If one name is chosen at random, what is the probability that it is Priya?

   (A) $\frac{1}{4}$

   (B) $\frac{1}{6}$

   (C) $\frac{2}{4}$

   (D) $\frac{2}{6}$

2. Select **all** of the words for which the probability of selecting the letter E at random is $\frac{1}{3}$.

   (A) THE

   (B) BEST

   (C) SNEEZE

   (D) FREES

   (E) SPEECH

3. Design a situation where the probability of one event is $\frac{1}{5}$ and another event is $\frac{1}{10}$. Explain your reasoning.

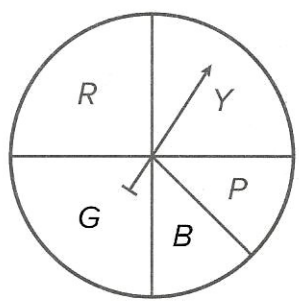
4. What is the probability of the spinner landing on the section labeled B? (Lesson 8-1)

   (A) $\frac{1}{8}$

   (B) $\frac{1}{5}$

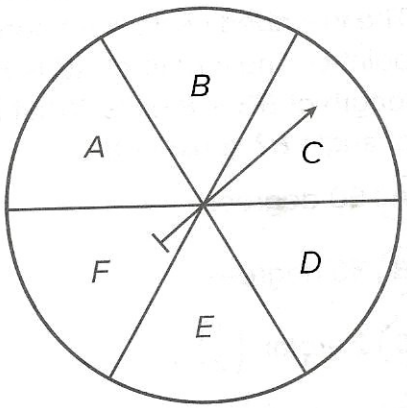
   (C) $\frac{1}{4}$

   (D) $\frac{1}{2}$



NAME _____ DATE _____ PERIOD _____

5. This spinner is spun 300 times. Estimate the number of times it would be expected to land on the section labeled B. (Lesson 8-1)

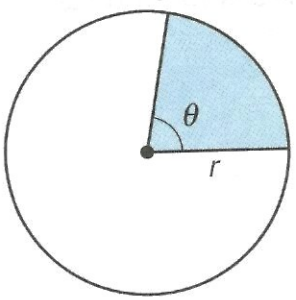


6. A circle has radius 5 units. For each angle measure, find the area of a sector of this circle with that central angle. (Lesson 7-13)

   a. π radians

   b. 3 radians

7. Select **all** formulas that could be used to find the area of this sector. The angle θ is measured in radians. (Lesson 7-13)



   (A.) $\frac{1}{2}r^2\theta$

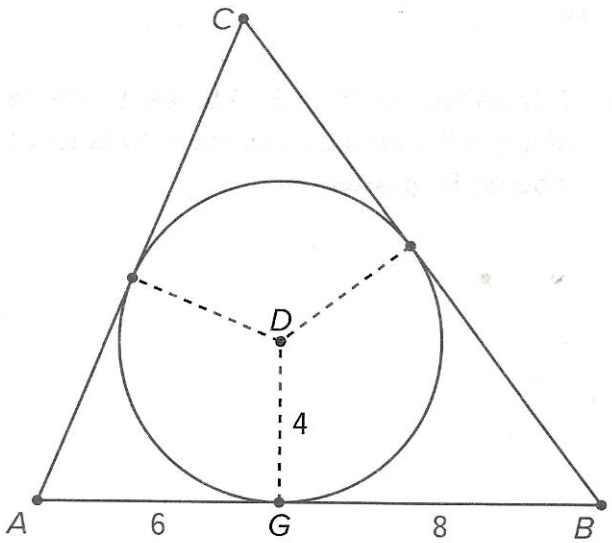
   (B.) $\frac{\theta}{2\pi} \cdot \pi r^2$

   (C.) $\frac{\theta}{360} \cdot \pi r^2$

   (D.) $\frac{\pi^2}{r} \cdot \theta$

   (E.) $\frac{\theta}{2\pi} \cdot 2\pi r$

8. Triangle ABC is shown with an inscribed circle of radius 4 units centered at point D. The inscribed circle is tangent to side AB at point G. The length of AG is 6 units and the length of BG is 8 units. What is the measure of angle B? (Lesson 7-7)



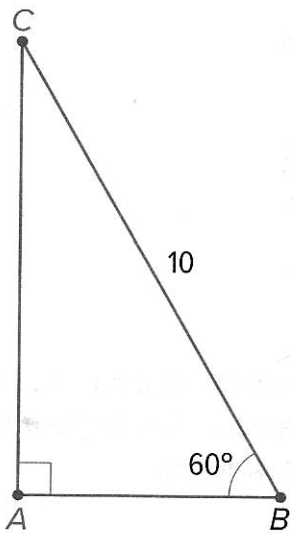
(A.) 60 degrees

(B.) 30 degrees

(C.) 2 arctan $\left(\frac{1}{2}\right)$

(D.) arctan $\left(\frac{1}{2}\right)$

9. Select **all** the true statements. (Lesson 4-3)



(A.) Angle C is 30 degrees.

(B.) Side AC is 5 units.

(C.) Side AB is 5 units.

(D.) Side AC is $5\sqrt{2}$ units.

(E.) Side AC is $10\sqrt{3}$ units.

Lesson 8-3

Sample Spaces

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's look closer at sample spaces.

Warm Up

3.1 Rolling Cubes

When rolling two standard number cubes, one of the possible outcomes is 1 and 1.

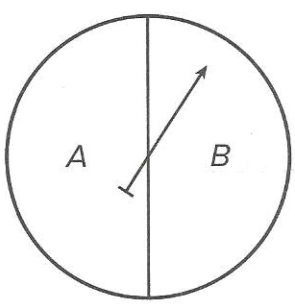
1. What are the other possible outcomes?

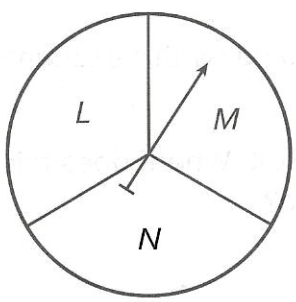
2. How many outcomes are in the sample space?

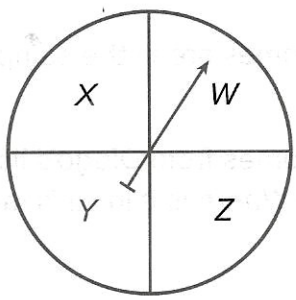
Activity

3.2 Spinner Sample Space

Each of the spinners is spun once.







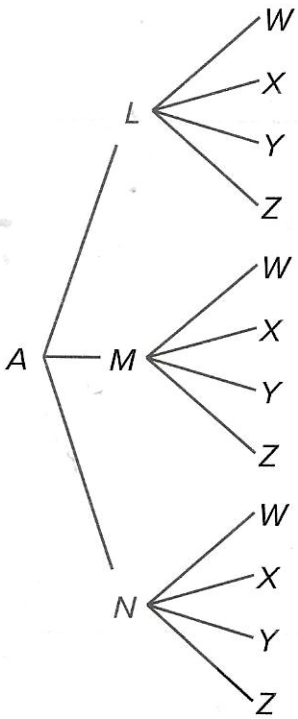
- Diego makes a list of the possible outcomes: ALW, ALX, ALY, ALZ, AMW, AMX, AMY, AMZ, ANW, ANX, ANY, ANZ, BLW, BLX, BLY, BLZ, BMW, BMX, BMY, BMZ, BNW, BNX, BNY, BNZ
- Tyler makes a table for the first two spinners.

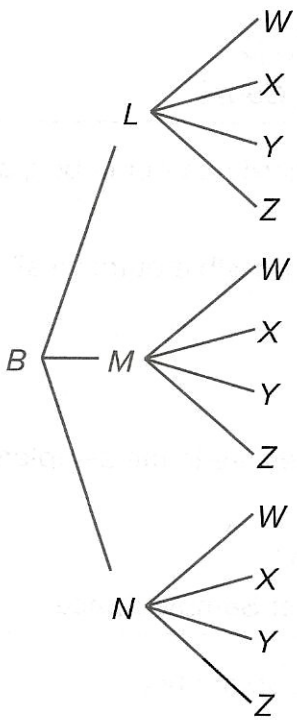
	L	M	N
A	AL	AM	AN
B	BL	BM	BN

Then he uses the outcomes from the table to include the third spinner.

	W	X	Y	Z
AL	ALW	ALX	ALY	ALZ
AM	AMW	AMX	AMY	AMZ
AN	ANW	ANX	ANY	ANZ
BL	BLW	BLX	BLY	BLZ
BM	BMW	BMX	BMY	BMX
BN	BNW	BNX	BNY	BNZ

- Lin creates a tree to keep track of the outcomes.





1. How many outcomes are in the sample space for this experiment?

2. One of the outcomes from Diego's list is BLX. Where does this show up in Tyler's method? Where is it in Lin's method?

3. When spinning all three spinners, what is the probability that:
   a. they point to the letters ANY? Explain your reasoning.

   b. they point to the letters AMW, ANZ, or BNW? Explain your reasoning.

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4. If a fourth spinner that has 2 equal sections labeled S and T is added, how would each of the methods need to adjust?

Activity

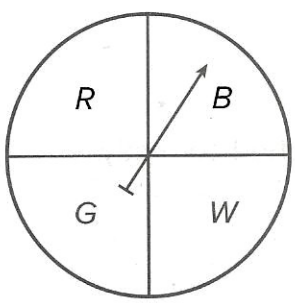
3.3 Sample Space Practice

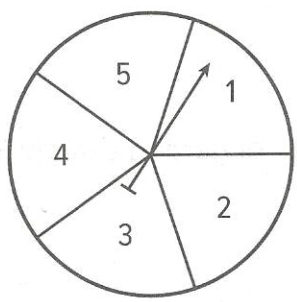
List all the possible outcomes for each experiment.

1. A standard number cube is rolled, then a coin is flipped.

2. Four coins are flipped.

3. The two spinners are spun.





4. A class block is chosen from 1, 2, 3, 4, or 5, then a subject is chosen from English or math.

Are you ready for more?

Elena is answering a matching practice problem where she has to match each of 4 items in a left column (A, B, C, D) to 4 items on the right (1, 2, 3, 4) so that each item is used exactly once. For example one way to answer this problem is A4, B2, C1, D3.

1. What are all the possible ways she could answer this problem?

2. The actual solution is A1, B2, C3, D4. If Elena was equally likely to guess any answer, use the sample space to find:
   a. The probability of getting exactly 0 items matched correctly.

   b. The probability of getting exactly 1 item matched correctly.

   c. The probability of getting exactly 2 items matched correctly.

   d. The probability of getting exactly 3 items matched correctly.

   e. The probability of getting exactly 4 items matched correctly.

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Summary

Sample Spaces

Probability represents the proportion of the time an event will occur when repeating an experiment many, many times. For complex experiments, the sample space can get very large very quickly, so it is helpful to have some methods for keeping track of the outcomes in the sample space.

In some cases, it makes sense to list all the outcomes in the sample space. For example, when flipping 3 coins, the 8 outcomes in the sample space are:

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

where H represents heads and T represents tails.

With more outcomes possible, it can be difficult to make sure all the outcomes are represented and none are repeated, so other methods may be helpful.

Another option is to use tables. When a complex experiment is broken down into parts, tables can be used to find the outcomes of two parts at a time. For example, when flipping 3 coins, we determine the outcomes for flipping just 2 coins.

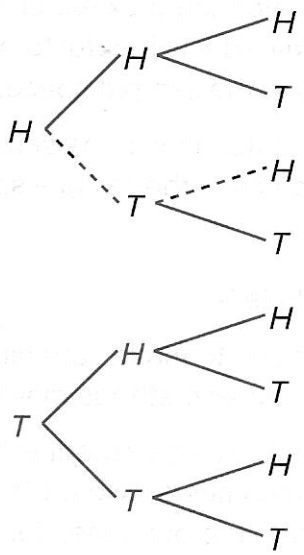
	H	T
H	HH	HT
T	TH	TT

The possible outcomes are represented by the 4 options in the middle of the table: HH, HT, TH, and TT. These outcomes can then be combined with the third coin flip in another table.

	H	T
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

Again, we see that the outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT.

Another way to keep track of the outcomes is to draw a tree structure. Each column represents another part of an experiment, with branches connecting each possible result from one part of the experiment to the possible results for the next part. By following the branches from left to right, each path represents an outcome for the sample space. The tree for flipping 3 coins would look like this:



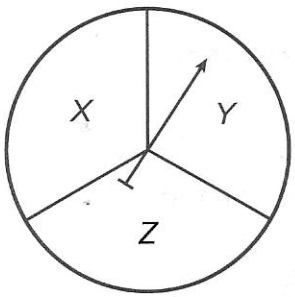
The path shown with the dashed line represents the HTH outcome. By following the other paths, the other 7 outcomes can be seen.

NAME _____ DATE _____ PERIOD _____

Practice

Sample Spaces

1. List all the possible outcomes for spinning the spinner and flipping a fair coin.



2. A student picks a random letter from the word "cat" and a random letter from the word "meow."

   a. How many outcomes are in the sample space?

   b. What is the probability that a "c" is chosen?

   c. What is the probability that a "w" is chosen?

   d. What is the probability that a "c" and a "w" are chosen?

3. Tyler decides which type of pizza to order. The choices for crust are thin crust or regular crust. The choices for one topping are pepperoni, mushrooms, olives, sausage, or green peppers. Tyler has trouble deciding because there are so many possibilities. He selects the type of crust and one topping at random. How many outcomes are in the sample space?

   (A.) 2

   (B.) 5

   (C.) 7

   (D.) 10

4. A spinner is divided into 5 equal sections. 2 of them are red, 1 of them is orange, 1 of them is yellow, and 1 of them is green. (Lesson 8-2)

   a. What is the probability that it lands on red?

   b. What is the probability that it lands on orange or yellow?

   c. What is the probability that it lands on blue?

5. Select **all** of the situations that have a 25% chance of occurring. (Lesson 8-1)

   (A.) Rolling a standard number cube and getting a 4.

   (B.) Flipping two fair coins and getting heads on the first flip and tails on the second flip.

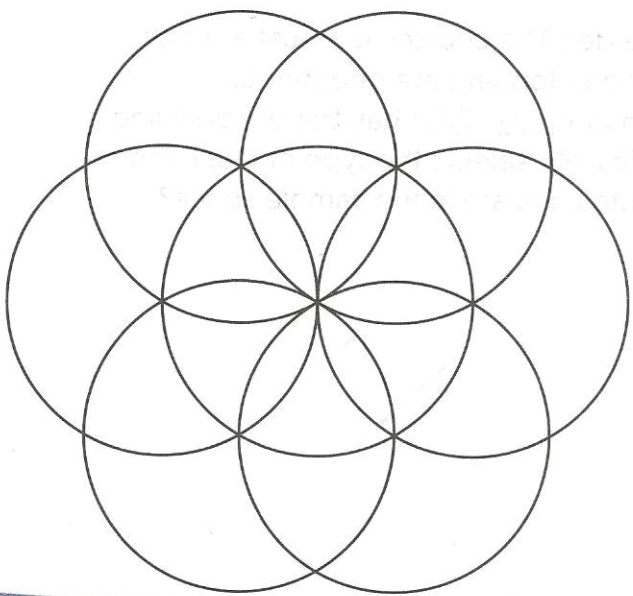
   (C.) Picking a letter at random from the word KALAMATA and getting an A.

   (D.) Picking a letter at random from the word CALAMITY and getting an A.

   (E.) Getting the correct answer when guessing randomly on a multiple choice question that has 4 choices.

6. A circle has radius 12 inches, and a central angle is drawn in. The length of the arc defined by the central angle is 20π inches. Find the area of the sector outlined by this arc. (Lesson 7-9)

7. Here is a straightedge and compass construction. Use a straightedge to draw a triangle that is *not* an equilateral triangle. Explain how you know. (Lesson 1-4)



Lesson 8-4

DATE _____ PERIOD _____

Tables of Relative Frequencies

NAME _____

Learning Goal Let's use tables to organize probabilities.

Warm Up

4.1 Notice and Wonder: Dog City

(l)Oleksandr Lytovnenko/ Shutterstock, (r)@Hans Surfer/Moment/Getty Images

This two-way table summarizes data from a survey of 200 people who reported their home environment (urban or rural) and pet preference (dog or cat).

	Urban	Rural	Total
Cat	54	42	96
Dog	80	24	104
Total	134	66	200

What do you notice? What do you wonder?

Activity

4.2 Rolling into Tables

Decide which person will be partner A and which will be partner B.

The result of partner A's roll is represented by the values on the left side of the table. The result of partner B's roll is represented by the values on the top of the table.

Roll your number cube. Record the result of the roll. For example, if partner A rolls a 3 and partner B rolls a 5, then record 3,5. Repeat this process until your teacher tells you to stop.

Use the table to summarize the results. For example, if 6,6 appears on your list a total of five times, write a 5 in the bottom right cell of the table.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

1. Do the values in the table match your expectation? Explain your reasoning.

2. Based on the table, how many times did partner A roll a 5?

3. How many times did you both roll the same number?

NAME _____ DATE _____ PERIOD _____

4. What percentage of the rolls resulted in the same number from both partners?

5. What percentage of the rolls resulted in partner A rolling a 3 and partner B rolling a 6?

6. Based on the table, estimate the probability that partner A will roll a 2 and partner B will roll a 4. Explain your reasoning.

Activity

4.3 Traveling Methods

1. A company has an office in Austin, Texas, and an office in Copenhagen, Denmark. The company wants to know how employees get to work, so they take a survey of all the employees and summarize the results in a table.

	Walk	Car	Public Transit	Bike	Total
Austin	63	376	125	63	627
Copenhagen	48	67	95	267	477
Total	111	443	220	330	1,104

   a. If an employee is selected at random, what is the probability that they work in Austin and drive a car to work?

   b. If an employee is selected at random, what is the probability that they work in Copenhagen and ride a bike to work?

   c. If an employee is selected at random, what is the probability that they take public transit to work?

   d. If an employee from Copenhagen is selected at random, what is the probability that they ride a bike to work?

   e. If an employee who takes public transit to work is selected at random, what is the probability they work in Austin?

   f. How are the last two questions different from the first three?

2. A school district is interested in how students get to school, so they survey their high school students to see how they get to school and separate the numbers by grade level. The results of the survey are summarized in the table.

	Car	Bus	Other Method	Total
Grade 9	1,141	3,196	228	4,565
Grade 10	1,126	1,770	322	3,218
Grade 11	1,732	799	133	2,664
Grade 12	1,676	447	111	2,234
Total	5,675	6,212	794	12,681

a. If a high school student is selected at random, what is the probability they are in grade 9 and ride the bus to school?

b. If a high school student is selected at random, what is the probability that they are in grade 12?

c. If a high school student is selected at random, what is the probability that they take a car to school?

d. If a grade 10 student is selected at random, what is the probability that they ride a bus to school?

e. If a grade 12 student is selected at random, what is the probability that they ride a bus to school?

f. If a student who rides the bus to school is selected at random, what is the probability that they are in grade 9?

Are you ready for more?

Clare surveyed 40 students at her school as part of a psychology project. Here are the two questions she asked:

- Do you like to swim? (Yes or No)
- What is your favorite season? (Winter, Spring, Summer, or Fall)

Here are the results of her survey.

	Likes to Swim	Does Not Like to Swim
Winter	5	4
Spring	8	3
Summer	11	1
Fall	4	4

1. Create a relative frequency table for Clare's data.

2. If a student that took Clare's survey is selected at random, what is the probability that they said they like to swim and their favorite season is summer?

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3. Create two of your own survey questions or use Clare's questions to survey 20 or more people. Record your survey questions and display the results in a table.

4. Create a relative frequency table for your own data. What is the most common response to your survey? What is the least common response to your survey?

Summary

Tables of Relative Frequencies

Tables provide a useful structure for organizing data. When several responses have been collected about some categorical variables, the data can be organized into a frequency table. The table can be used to calculate relative frequencies, which can be interpreted as probabilities.

For example, 243 participants in a survey responded to questions about their favorite season and whether they like wearing pants or shorts better. The results are summarized in the table.

	Pants	Shorts
Winter	21	16
Spring	43	20
Summer	18	56
Autumn	40	29

This table can be turned into a relative frequency table by dividing each of the values in the cells by the total number of participants.

	Pants	Shorts
Winter	0.09	0.07
Spring	0.18	0.08
Summer	0.07	0.23
Autumn	0.16	0.12

If a person is randomly selected from among these 243 participants, we can see that the probability that the chosen person's favorite season is spring and likes shorts better than pants is 0.08. We can also use the fact that there were 63 people who listed spring as their favorite season (43 + 20), so the probability that a randomly selected person from this group likes spring best is around 0.26 $\left(\frac{63}{243} \approx 0.26\right)$.

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Practice

Tables of Relative Frequencies

1. The table shows the results from a survey that asked 100 adults if they had a high school diploma and if their annual income was more than $30,000.

	$30,000 or Less	More than $30,000
High School Diploma	21	68
No High School Diploma	9	2

A person who took the survey is selected at random.

a. What is the probability that the person has a high school diploma and makes $30,000 or less?

b. What is the probability that the person has no high school diploma and earns more than $30,000?

2. The table shows data from a science fair experiment that studied the number of eggs that hatched at three different temperatures.

	Cool	Room Temperature	Warm
Hatched	6	14	23
Not Hatched	19	11	2

a. What percentage of the eggs hatched?

b. What percentage of the eggs that were at the cool temperature hatched?

c. What percentage of the eggs were not at room temperature?

d. What percentage of the eggs were at the warm temperature and did not hatch?

3. The table shows information from a survey about the resting heart rate in beats per minute (bpm), for 50 people living at altitudes above and below 10,000 feet.

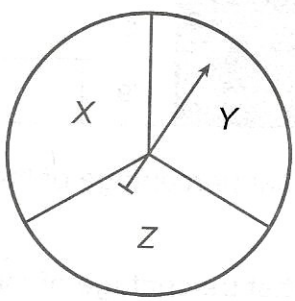
	Below 80 bpm	Above 80 bpm	Total
Above 10,000 ft	3	19	22
10,000 ft or Below	16	12	28
Total	19	31	50

a. Create a two-way table that shows the relative frequency for each of the values in the table relative to all 50 people in the survey.

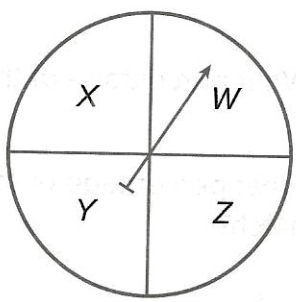
b. What is the probability that a person surveyed, selected at random, has a heart rate above 80 bpm or lives above 10,000 ft?

c. What is the probability that a person surveyed, selected at random, has a heart rate above 80 bpm and lives above 10,000 ft?

4. List all the possible outcomes for spinning the spinner and rolling a fair number cube. **(Lesson 8-3)**



5. A student flips a fair coin and then spins this spinner. How would you find the sample space? **(Lesson 8-3)**



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6. Select **all** of the words for which the probability of selecting the letter A at random is $\frac{1}{4}$. (Lesson 8-2)

   (A.) AREA

   (B.) ACID

   (C.) ANGRY

   (D.) APPEASED

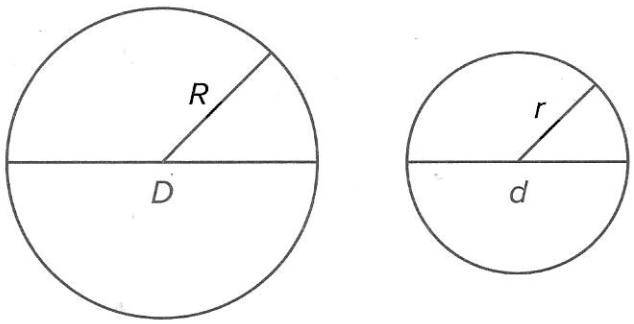
   (E.) AARDVARK

7. On an assignment, there are two multiple choice questions with four answers choices each. You have no idea what the correct answer is to either one so you guess. (Lesson 8-1)

   a. What is the probability that you get both of them right by guessing? Explain your answer.

   b. What is the probability that you get exactly one of them right by guessing? Explain your answer.

8. Here are 2 circles. The smaller circle has radius r, circumference c, and diameter d. The larger circle has radius R, circumference C, and diameter D. The larger circle is a dilation of the smaller circle by a factor of k.



Using the circles, match the pairs of ratios with their values. (Lesson 7-10)

A. The ratios $\frac{C}{c}$ and $\frac{R}{r}$ are both equal to _____.

B. The ratios $\frac{C}{D}$ and $\frac{c}{d}$ are both equal to _____.

C. The ratios $\frac{D}{R}$ and $\frac{d}{r}$ are both equal to _____.

1. 2
2. k
3. π

Lesson 8-5

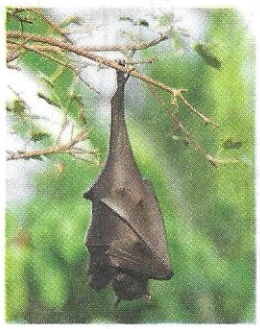
Combining Events

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's look at ways to describe events composed of other events.

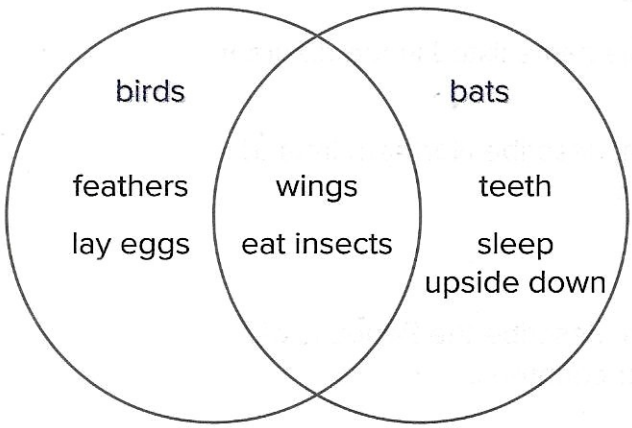
Warm Up

5.1 Notice and Wonder: Birds and Bats



(l)steve_byland/123RF, (r) Christian Rummel/iStock/Getty Images

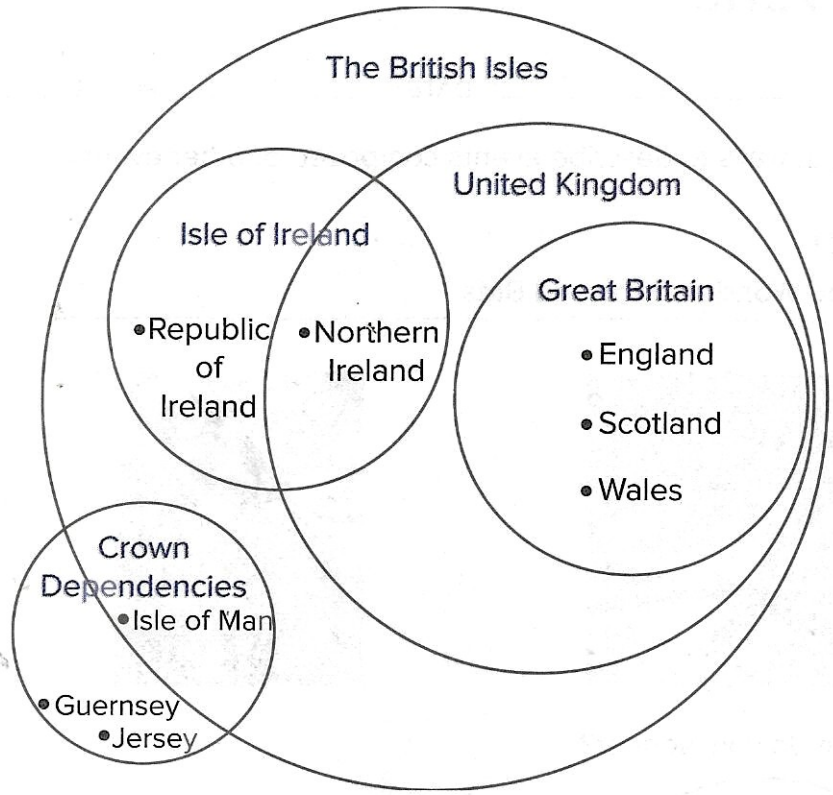
What do you notice? What do you wonder?



6 legs

Activity

5.2 Eventful Islands



The small dots next to the names indicate that the name listed in the diagram is a country.

1. Based on the categories in the Venn diagram, describe Northern Ireland in a way that will not include any other countries.

2. Based on the categories in the Venn diagram, describe the Republic of Ireland in a way that will not include any other countries.

3. How many countries displayed are not part of The British Isles?

4. How many countries displayed are part of the United Kingdom?

5. How many countries displayed are part of the Isle of Ireland?

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6. How many places displayed are part of the United Kingdom and the Isle of Ireland?

7. How many places displayed are part of the United Kingdom or the Isle of Ireland?

8. If one of the crown dependencies (there are 3) is chosen at random, what is the probability that it is part of The British Isles?

9. Northern Ireland, England, Scotland and Wales are all part of the United Kingdom. If one of them is selected at random, what is the probability that it is also considered part of Great Britain?

10. Given that the Republic of Ireland, Northern Ireland, England, Scotland, Wales, and the Isle of Man are all part of The British Isles, what is the probability that one of them selected at random is part of the Island of Ireland?

Activity

5.3 Info Gap: College and Career Planning

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

Activity

5.4 Number Cube Descriptions

1. Roll two standard number cubes twenty times. Record your results in the table.

First Cube	Second Cube

First Cube	Second Cube

2. List all the possible outcomes in each event. The first one is done for you.

   a. The first cube is a 6. 6,1; 6,2; 6,3; 6,4; 6,5; 6,6

   b. The cubes have a 4 and a 6.

   c. The cubes are doubles. (Doubles means that the number on the two number cubes is the same.)

   d. The cubes are doubles and the first cube is a 6.

   e. The cubes are doubles or the first cube is a 6.

   f. The first cube is not a 6.

   g. The cubes are doubles and the first cube is not a 6.

   h. The cubes are not doubles.

NAME _____ DATE _____ PERIOD _____

3. Use the information in the table to answer the questions.

   a. What percentage of the rolls have a 6 on the first cube?

   b. What percentage of the rolls have a 4 and a 6?

   c. What percentage of the rolls are doubles?

   d. What percentage of the rolls are doubles and have a 6 on the first cube?

   e. What percentage of the rolls are doubles or have a 6 on the first cube?

   f. What percentage of the rolls do not have a 6 on the first cube?

   g. What percentage of the rolls are doubles and do not have a 6 on the first cube?

   h. What percentage of the rolls are not doubles?

4. The sample space has 36 outcomes. Use this and the number of outcomes in each event to find the actual probability for each event in the previous problem. Compare your answers.

5. Why is the actual probability different from the percentage of rolls you made for each event?

Are you ready for more?

1. Roll three standard number cubes forty times. Record your results in the table.

First Cube	Second Cube	Third Cube	First Cube	Second Cube	Third Cube

NAME _____ DATE _____ PERIOD _____

2. List all the possible outcomes in each event. The first one is done for you.

   a. The first cube is a 6. 661, 662, 663, 664, 665, 666, 651, 652, 653, 654, 655, 656, 641, 642, 643, 644, 645, 646, 631, 632, 633, 634, 635, 636, 621, 622, 623, 624, 625, 626, 611, 612, 613, 614, 615, 616

   b. The cubes have a 4, a 5, and a 6.

   c. The cubes are triples. (Triples means that the number on the three number cubes is the same.)

3. Use the information in the table to answer the questions.

   a. What percentage of the rolls have a 6 on the first cube?

   b. What percentage of the rolls have a 4, a 5, and a 6?

   c. What percentage of the rolls are triples?

4. The sample space has 216 outcomes. Use this and the number of outcomes in each event to find the actual probability for each event in the previous problem. Compare your answers.

Are you ready for more?

5. If you rolled 500 times, do you think that the difference between the actual probability and the percentage of rolls you made for each event would increase or decrease? Explain your reasoning.

6. Describe a method for recording the 216 outcomes in the sample space. Do not actually record all 216 outcomes.

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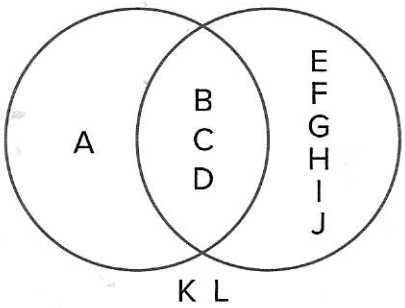
Summary

Combining Events

In many cases, it is useful to talk about the important outcomes in a sample space by naming characteristics the outcomes share or characteristics that are not in the outcomes.

For example, consider a group of 12 first-year students in college and their choices of science courses.



The circle on the left represents the students taking a chemistry class and the circle on the right represents the students taking a biology class. The region where the circles overlap represents the students taking both a chemistry class and a biology class. The students who are not included in either circle are not taking chemistry or biology.

We can describe some of the groups of students based on the characteristics they share or lack. For example:

- students A, B, C, and D are taking a chemistry course
- students A, K, and L are not taking a biology course
- students B, C, and D are taking a chemistry course and taking a biology course
- students E, F, G, H, I, and J are taking a biology course and are not taking a chemistry course
- students A, B, C, D, E, F, G, H, I, and J are taking a chemistry course or are taking a biology course

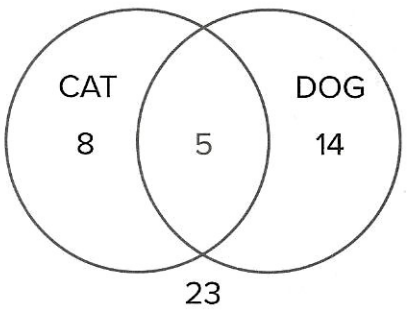
While listing the individual students in this case is not too difficult, when there are many outcomes in a sample space (which is often the case), it is often easier to describe the interesting ones using some characteristics they share.

Practice

Combining Events

1. 50 students were asked two survey questions:
   - Do you have a dog?
   - Do you have a cat?

   Their responses are summarized in the Venn diagram.



   a. How many students have a cat?

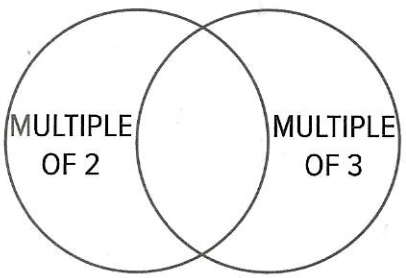
   b. How many students have a cat or a dog?

   c. How many students have a cat and a dog?

   d. How many students do not have a cat?

   e. How many students do not have a cat or a dog?

2. In the Venn diagram, the circle on the left represents all the whole numbers between 1 and 12 that are multiples of 2. The circle on the right represents all the numbers between 1 and 12 that are multiples of 3.



   a. Which numbers are multiples of 2?

   b. Which numbers are multiples of 3?

   c. Which numbers belong in the region where the two circles overlap? Explain your reasoning.

   d. Which whole numbers between 1 and 12 are not contained inside either circle in the Venn diagram?

   e. What is the probability that a whole number between 1 and 12, selected at random, is a multiple of two or three?

   f. What is the probability that a whole number between 1 and 12, selected at random, is not a multiple of 2?

NAME _____ DATE _____ PERIOD _____

3. Two classes of elementary school students are going on a field trip, and they will be provided with a snack. Each student selects one snack option. The table summarizes the snack preference of each student in the class.

	Apple	Carrot Sticks	Peach Slices
Class A	4	6	12
Class B	7	3	14

a. What is the probability that a student selected at random prefers peach slices as a snack?

b. What is the probability that a student selected at random prefers an apple or carrot sticks as a snack?

c. What is the probability that a student in class A selected at random prefers an apple as a snack?

d. What is the probability that a student selected at random is in class B and prefers carrot sticks as a snack?

4. The table shows the results from a survey that asked 200 adults if they had a college diploma and if their annual income was more than $40,000.

	$40,000 or Less	More Than $40,000
College Diploma	44	101
No College Diploma	27	28

A person who took the survey is selected at random. (Lesson 8-4)

a. What is the probability that the person has a college diploma and makes $40,000 or less?

b. What is the probability that the person doesn't have a college diploma and earns more than $40,000?

5. The table shows data from a science fair experiment that studied the average growth rate of 20 samples of fungus at 70 degrees Fahrenheit and 20 samples of fungus at 80 degrees Fahrenheit. (Lesson 8-4)

	70 Degrees	80 Degrees
Above Average Growth Rate	3	8
Average Growth Rate	12	11
Below Average Growth Rate	5	1

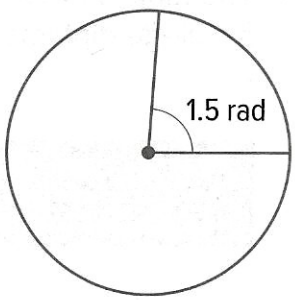
a. What percentage of the samples had an above average growth rate?

b. What percentage of the samples at 70 degrees had an above average growth rate?

c. What percentage of the samples were at 80 degrees?

d. What percentage of the samples that had an average growth rate were at 80 degrees?

6. Here is a central angle that measures 1.5 radians.



Select the statement that *must* be true. (Lesson 7-11)

(A.) The area of the whole circle is 1.5 times the area of the slice.

(B.) The circumference of the whole circle is 1.5 times the length of the arc formed by the angle.

(C.) The length of the arc defined by the angle is 1.5 times longer than the radius.

(D.) The length of the arc is 1.5π units.

Lesson 8-6

The Addition Rule

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's learn about and use the addition rule.

Warm Up

6.1 Hats Off, Sneakers On

The table displays information about people at a neighborhood park.

	Wearing Sneakers	Not Wearing Sneakers	Total
Wearing a Hat	8	2	10
Not Wearing a Hat	3	12	15
Total	11	14	25

1. Andre says the number of people wearing sneakers or wearing a hat is 21, because there are a total of 10 people wearing a hat and a total of 11 people wearing sneakers. Is Andre correct? Explain your reasoning.

2. What is the probability that a person selected at random from those in the park is wearing sneakers or wearing a hat?

Activity

6.2 State Names

Jada has a way to find the probability of a random outcome being in event A or event B. She says, "We use the probability of the outcome being in event A, then add the probability of the outcome being in category B. Now some outcomes have been counted twice, so we have to subtract the probability of the outcome being in both events so that those outcomes are only counted once."

Jada's method can be rewritten as:

$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$

1. The table of data summarizes information about the 50 states in the United States from a census in the year 2000. A state is chosen at random from the list of 50. Let event A be "the state name begins with A through M" and event B be "the population of the state is less than 4 million."

	Population Less Than 4 Million	Population at Least 4 Million
Name Begins with A Through M	11	15
Name Begins with N Through Z	13	11

Alaska is one of the 11 states in the top left cell of the table. California is one of the 15 states in the top right cell of the table. Nebraska is one of the 13 states in the bottom left cell of the table. New York is one of the 11 states in the bottom right cell of the table. For each event, write which of the four states listed here is an outcome in that event.

a. A or B

b. A

c. B

d. A and B

2. Find each of the probabilities when a state is chosen at random:

a. P(A or B)

b. P(A)

c. P(B)

d. P(A and B)

3. Does Jada's formula work for these events? Show your reasoning.

4. Seniors at a high school are allowed to go off campus for lunch if they have a grade of A in all their classes or perfect attendance. An assistant principal in charge of academics knows that the probability of a randomly selected senior having A's in all their classes is 0.1. An assistant principal in charge of attendance knows that the probability of a randomly selected senior having perfect attendance is 0.16. The cafeteria staff know that the probability of a randomly selected senior being allowed to go off campus for lunch is 0.18. Use Jada's formula to find the probability that a randomly selected senior has all As and perfect attendance.

NAME _____ DATE _____ PERIOD _____

Are you ready for more?

Priya lists all of the multiples of 3 for whole numbers between 1 and 48 inclusive.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48

Tyler lists all of the multiples of 4 between 1 and 48 inclusive.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

1. Use a Venn diagram to display the multiples of 3 and the multiples of 4 between 1 and 48.

2. If a whole number between 1 and 48 inclusive is selected at random, find each probability:

   a. P(multiple of 3)

   b. P(multiple of 4)

   c. P(multiple of 3 or 4)

   d. P(multiple of 3 and 4)

3. Priya and Tyler extend their lists of multiples to include whole numbers between 1 and 96 inclusive. If a whole number between 1 and 96 inclusive is selected at random, find each probability and compare it to the probability in the previous problem:

   a. P(multiple of 3)

   b. P(multiple of 4)

   c. P(multiple of 3 or 4)

   d. P(multiple of 3 and 4)

4. If a whole number between 1 and 100 inclusive is selected at random, what do you think P(multiple of 3 and 4) is? Explain your reasoning.

5. If a whole number between 1 and n, inclusive, for any whole number value of n, what is P(multiple of 3 and 4)?

Activity

6.3 Coffee or Juice?

1. At a cafe, customers order coffee at the bar, and then either go to another table where the cream and sugar are kept, or find a seat. Based on observations, a worker estimates that 70% of customers go to the second table for cream or sugar. The worker also observes that about 60% of all customers use cream for their coffee and 50% of all customers use sugar. Use the worker's estimates to find the percentage of all customers who use both cream and sugar for their coffee. Explain or show your reasoning.

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2. At the grocery store, 70% of the different types of juice come in a bottle holding at least 400 milliliters (mL) and 40% of the different types of juice come in a low-sugar version. Only 25% of the juice varieties are in bottles holding at least 400 mL and have a low-sugar version. What percentage of the different types of juice come in a bottle holding at least 400 mL or are low-sugar? Explain your reasoning.

3. Complete the table showing the number of cans and bottles of low-sugar and regular juice in one of the shelves at a grocery store. You may not use zero in any of the empty spaces.

	Less Than 400 mL	At Least 400 mL	Total
Low-Sugar Available			80
No Low-Sugar Available			
Total	60	140	

Use the table to find the probabilities for a juice chosen from the shelf at random.

a. *P*(less than 400 mL)

b. *P*(no low-sugar available)

c. *P*(no low-sugar available or less than 400 mL)

d. *P*(no low-sugar available and less than 400 mL)

Summary

The Addition Rule

The **addition rule** is used to compute probabilities of compound events. The addition rule states that, given events A and B, $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$.

For example, the student council sold 100 shirts that are either gray or blue and in sizes medium and large.

	Medium	Large	Total
Gray	20	10	30
Blue	15	55	70
Total	35	65	100

A student who bought a shirt is chosen at random. One way to find the probability that this student bought a shirt that is blue or medium is to begin with the probabilities for shirts sold in that color and size. The probability that the student bought a blue shirt is 0.70 since 70 out of the 100 shirts sold were blue. The probability that the student bought a medium shirt is 0.35 since 35 out of the 100 shirts sold were medium.

Because we are interested in the probability of a blue shirt or a medium shirt being purchased, we might think we should add these probabilities together to find $0.70 + 0.35 = 1.05$.

This doesn't seem to work since it is saying the probability is greater than 1.

The problem is that the 15 students who bought shirts that are both medium in size and blue are counted twice when we do this. To fix this double counting, we should subtract the probability that the chosen student is in both categories so that these students are only counted once.

The addition rule then shows that the probability the student bought a medium shirt or a blue shirt is 0.90 since $P(\text{medium or blue}) = P(\text{medium}) + P(\text{blue}) - P(\text{medium and blue})$ or $P(\text{medium or blue}) = 0.35 + 0.70 - 0.15$, which is 0.90.

Glossary

addition rule

NAME _____ DATE _____ PERIOD _____

Practice

The Addition Rule

1. The table displays the number and type of tickets bought for a play that was performed in both the afternoon and evening.

	Child Ticket	Adult Ticket
Afternoon	36	127
Evening	12	188

The addition rule states that given events A and B, $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$. Show how to apply the addition rule for the events "a child ticket is bought for a play" and "the play is performed in the afternoon."

2. A biologist studies two different invasive species, purple loosestrife and the common reed, at sites in both wetland and coastal habitats. Purple loosestrife is present in 35% of the sites. Common reed is present in 55% of the sites. Both purple loosestrife and common reed are present in 23% of the sites. What percentage of the sites have purple loosestrife or common reed present?

(A.) 12%

(B.) 22%

(C.) 67%

(D.) 90%

3. 30 teachers and students participate in a student-faculty basketball game as a fundraiser. They are surveyed after the game by the sports medicine class to find out how many of them stretched before the game. The results of the survey are shown in the table.

	Younger Than 18 Years Old	18–30 Years Old	More Than 30 Years Old
Stretched Before the Game	5	9	4
Did not Stretch Before the Game	10	1	1

One of the sports medicine students, Han, wants to know the probability that one of the participants in the game selected at random is younger than 18 years old or stretched before the game. To figure this out, he adds the three values in the first row of the table (5, 9, and 4) to the two values listed under the heading "younger than 18 years old" (5 and 10). He then divides that answer by 30 and obtains a probability of $\frac{33}{30}$. Han realizes that $\frac{33}{30}$ is greater than 1 and determines that he must have made a mistake.

a. What is Han's mistake? Explain your reasoning.

b. How does the addition rule account for this kind of mistake?

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4. Two classes of elementary school students were asked what activity they want to do during recess. Each student selects one recess option. The table summarizes the recess preference of each student in the class. (Lesson 8-5)

	Tag	Playground	Kickball
Class A	2	7	9
Class B	7	10	5

a. What is the probability that a student selected at random prefers to play tag at recess?

b. What is the probability that a student selected at random prefers to play tag or kickball at recess?

c. What is the probability that a student in class A selected at random prefers to play kickball at recess?

d. What is the probability that a student selected at random is in class B and prefers to play tag at recess?

5. A student picks a random letter from the word "dog" and a random letter from the word "barks." (Lesson 8-3)

a. How many outcomes are in the sample space?

b. What is the probability that an "o" is chosen?

c. What is the probability that a "k" is chosen?

d. What is the probability that a "o" and a "k" are chosen?

6. The table shows information from a survey about the resting heart rate in beats per minute (bpm), for 200 college students who are in the marching band and who are not in the marching band. (Lesson 8-4)

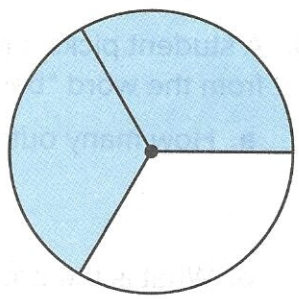
	Below 80 bpm	Above 80 bpm	Total
In the Marching Band	56	28	84
Not in the Marching Band	66	50	116
Total	122	78	200

a. Create a two-way table that shows the relative frequency for each of the values in the table relative to all 200 people in the survey.

b. What is the probability that a person surveyed, selected at random, has a heart rate above 80 bpm or is in the marching band?

c. What is the probability that a person surveyed, selected at random, has a heart rate below 80 bpm and is not in the marching band?

7. The circle in the image has been divided into congruent sectors. What is the measure of the central angle of the shaded region in radians? (Lesson 7-12)



Lesson 8-7

Related Events

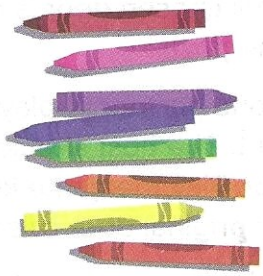
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's see how events are related.

Warm Up

7.1 Drawing Crayons

A bag contains 1 crayon of each color: red, orange, yellow, green, blue, pink, maroon, and purple.



1. A person chooses a crayon at random out of the bag, uses it for a bit, then puts it back in the bag. A second person comes to get a crayon chosen at random out of the bag. What is the probability the second person gets the yellow crayon?

2. A person chooses a crayon at random out of the bag and walks off to use it. A second person comes to get a crayon chosen at random out of the bag. What is the probability the second person gets the yellow crayon?

Activity

7.2 Choosing Doors

1. On a game show, a contestant is presented with 3 doors. One of the doors hides a prize and the other two doors have nothing behind them.
   - The contestant chooses one of the doors by number.
   - The host, knowing where the prize is, reveals one of the empty doors that the contestant did not choose.
   - The host then offers the contestant a chance to stay with the door they originally chose or to switch to the remaining door.
   - The final chosen door is opened to reveal whether the contestant has won the prize.

   Choose one partner to play the role of the host and the other to be the contestant. The host should think of a number: 1, 2, or 3 to represent the prize door. Play the game keeping track of whether the contestant stayed with their original door or switched and whether the contestant won or lost.

   Switch roles so that the other person is the host and play again. Continue playing the game until the teacher tells you to stop. Use the table to record your results.

	Stay	Switch	Total
Win			
Lose			
Total			

   a. Based on your table, if a contestant decides they will choose to stay with their original choice, what is the probability they will win the game?

   b. Based on your table, if a contestant decides they will choose to switch their choice, what is the probability they will win the game?

   c. Are the two probabilities the same?

NAME _____ DATE _____ PERIOD _____

2. In another version of the game, the host forgets which door hides the prize. The game is played in a similar way, but sometimes the host reveals the prize and the game immediately ends with the player losing, since it does not matter whether the contestant stays or switches.

Choose one partner to play the role of the host and the other to be the contestant. The contestant should choose a number: 1, 2, or 3. The host should choose one of the other two numbers. The contestant can choose to stay with their original number or switch to the last number.

After following these steps, roll the number cube to see which door contains the prize:

- Rolling 1 or 4 means the prize was behind door 1.
- Rolling 2 or 5 means the prize was behind door 2.
- Rolling 3 or 6 means the prize was behind door 3.

Play the game keeping track of whether the contestant stayed with their original door or switched and whether the contestant won or lost.

Switch roles so that the other person is the host and play again. Continue playing the game until the teacher tells you to stop. Use the table to record your results.

	Stay	Switch	Total
Win			
Lose			
Total			

a. Based on your table, if a contestant decides they will choose to stay with their original choice, what is the probability they will win the game?

b. Based on your table, if a contestant decides they will choose to switch with their original choice, what is the probability they will win the game?

c. Are the two probabilities the same?

Are you ready for more?

In another version of the game, the contestant is presented with 5 doors. One of the doors hides a prize and the other four doors have nothing behind them.

- The contestant chooses 3 doors by number.
- The host, knowing where the prize is, reveals 3 of the doors that have nothing behind them. Two of the doors that the contestant has chosen that are empty and one of the other doors that are empty.
- The host then offers the contestant a chance to stay with the door they originally chose or to switch to the remaining door.
- The final chosen door is opened to reveal whether the contestant has won the prize.

Choose one partner to play the role of the host and the other to be the contestant. The host should think of a number: 1, 2, 3, 4, or 5 to represent the prize door. Play the game keeping track of whether the contestant stayed with their original door or switched and whether the contestant won or lost.

Switch roles so that the other person is the host and play again. Continue playing the game until the teacher tells you to stop. Use the table to record your results.

	Stay	Switch	Total
Win			
Lose			
Total			

1. Based on your table, if a contestant decides they will choose to stay with their original choice, what is the probability they will win the game?

2. Based on your table, if a contestant decides they will choose to switch with their original choice, what is the probability they will win the game?

3. Are the two probabilities the same?

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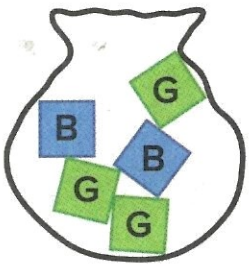
Summary

Related Events

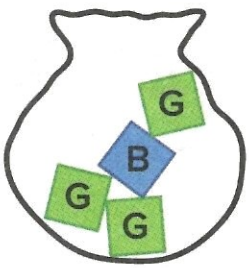
When considering probabilities for two events it is useful to know whether the events are independent or dependent. **Independent events** are two events from the same experiment for which the probability of one event is not affected by whether the other event occurs or not. **Dependent events** are two events from the same experiment for which the probability of one event *is* affected by whether the other event occurs or not.



For example, let's say a bag contains 3 green blocks and 2 blue blocks. You are going to take two blocks out of the bag.

Consider two experiments:

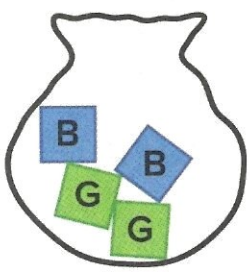
1. Take a block out, write down the color, return the block to the bag, and then choose a second block. The event, "the second block is green" is independent of the event, "the first block is blue." Since the first block is replaced, it doesn't matter what block you picked the first time when you pick a second block.
2. Take a block out, hold on to it, then take another block out. The same two events, "the second block is green" and "the first block is blue," are dependent.



If you get a blue block on the first draw, then the bag has 3 green blocks and 1 blue block in it, so $P(\text{green}) = \frac{3}{4}$.

If you get a green block on the first draw, then the bag has 2 green blocks and 2 blue blocks in it, so $P(\text{green}) = \frac{1}{2}$.

Since the probability of getting a green block on the second draw changes depending on whether the event of drawing a blue block on the first draw occurs or not, the two events are dependent.



In some cases, it is difficult to know whether events are independent without collecting some data. For example, a basketball player shoots two free throws. Does the probability of making the second shot depend on the outcome of the first shot? Some data would need to be collected about how often the player makes the second shot overall and how often the player makes the second shot after making the first so that you could compare the estimated probabilities.

Glossary

dependent events
independent events

Practice

Related Events

1. Each of the letters A through J are printed on tiles that are placed in a hat. Andre selects a tile at random and then replaces it. Clare then selects a tile at random.
   a. What is the probability that Andre selects a tile labeled B?
   b. What is the probability that Clare selects a tile labeled B?
   c. What is the probability that both Andre and Clare select a tile labeled B?
   d. Are the events of Andre selecting a tile and Clare selecting a tile dependent or independent? Explain your reasoning.

2. The Bulldogs have won approximately 67% of their 30 baseball games this season. The Bulldogs won 9 of the 10 games they played when Diego was the starting pitcher. Are the events "the Bulldogs win the game" and "the Bulldogs win the game when Diego is the starting pitcher" dependent or independent events? Explain your reasoning.

3. Describe two events that are independent. Explain your reasoning.

NAME _____ DATE _____ PERIOD _____

4. The table displays the number and type of tickets bought for a baseball tournament held on a Saturday and Sunday.

	Child Ticket	Adult Ticket
Saturday	27	41
Sunday	14	29

The addition rule states that given events A and B, $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$. Show how to apply the addition rule for the events "a child ticket is bought for a baseball tournament" and "the day of the tournament is Sunday." (Lesson 8-6)

5. A researcher surveys 200 randomly selected college students to determine the number of education majors and mathematics majors. 16% of the students surveyed are education majors and 6% of the students surveyed are mathematics majors. 2% of the students are both education majors and mathematics majors. What percentage of the students surveyed are education majors or mathematics majors? (Lesson 8-6)

(A.) 2%

(B.) 20%

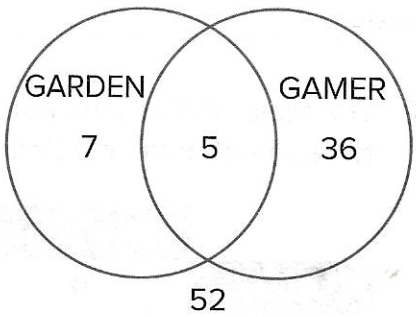
(C.) 22%

(D.) 24%

6. 100 students were asked two survey questions:

   - Do you have a garden?
   - Are you a gamer?

   Their responses are summarized in the Venn diagram.



   How many students surveyed have a garden? (Lesson 8-5)

   (A) 5

   (B) 7

   (C) 12

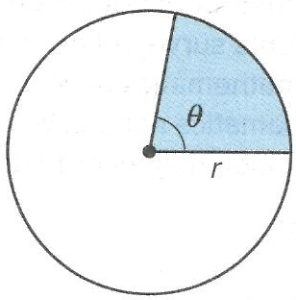
   (D) 36

7. The table shows the preferences of 40 students surveyed about the design of a new t-shirt for students who are graduating this year. (Lesson 8-4)

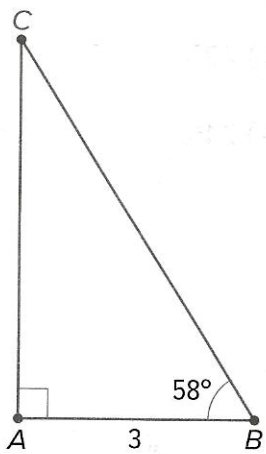
	Short Sleeves	Long Sleeves
Pink	12	5
Black	14	9

   What is the probability that a student surveyed, selected at random, preferred a pink t-shirt with long sleeves?

8. Write a formula that could be used to find the area of this sector. (Lesson 7-13)



9. *Technology required.* Find the missing measurements in triangle *ABC*. (Lesson 4-7)



Lesson 8-8

Conditional Probability

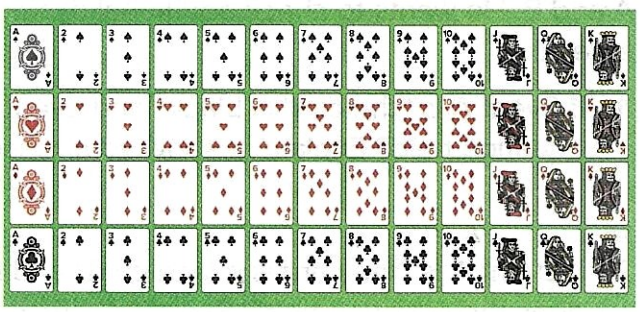
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's examine conditional probability.

Warm Up

8.1 She Made Some Tarts

1. Noah will select 1 card at random from a standard deck of cards. Find the probabilities. Explain or show your reasoning.



   a. *P*(the card is a queen)

   b. *P*(the card is a heart)

   c. *P*(the card is a queen and heart)

2. Elena pulls out only the hearts from the deck and sets the rest of the cards aside. She shuffles the hearts and draws one card. What is the probability she gets a queen?

Activity

8.2 Under One Condition

Kiran notices that the probabilities from the warm-up can be arranged into at least two equations.

P(the card is a queen and heart) = P(the card is a queen | the card is a heart) • P(the card is a heart) since $\frac{1}{52} = \frac{1}{13} \cdot \frac{13}{52}$.

P(the card is a queen and heart) = P(the card is a heart | the card is a queen) • P(the card is a queen) since $\frac{1}{52} = \frac{1}{4} \cdot \frac{4}{52}$.

Kiran wonders if it is always true that P(A and B) = P(A | B) • P(B) for events A and B. He wants to check additional examples from drawing a card from a deck.

1. If Event A is "the card is black" and Event B is "the card is a king," does the equation hold? Explain or show your reasoning.

2. If Event A is "the card is a face card" and Event B is "the card is a spade," does the equation hold? Explain or show your reasoning.

Activity

8.3 Coin and Cube

A coin is flipped, then a standard number cube is rolled. Let A represent the event "the coin lands showing heads" and B represent "the standard number cube lands showing 4."

1. Are events A and B independent or dependent? Explain your reasoning.

2. Find the probabilities.

   a. P(A)

   b. P(B)

   c. P(A | B)

   d. P(B | A)

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3. Describe the meaning of the events "not A" and "not B" in this situation, then find the probabilities.

   a. $P(\text{A} \mid \text{not B})$

   b. $P(\text{B} \mid \text{not A})$

4. Are any of the probabilities the same? Is there a relationship between those situations? Explain your reasoning.

Are you ready for more?

Students are writing programs in their robotics class using two different programming languages that are very similar. 70% of the programs are written in the first programming language and the other 30% percent are written in the second programming language. The newer robots only accept programs written in the first programming language and the older robots only accept programs written in the second programming language.

In this robotics class, Andre's job is to determine which programming language each program is written in. He determines the programming language correctly 90% of the time for each of the programs. If Andre determines that a program is written in the second programming language, what is the probability that the program is actually written in the first programming language? Explain your reasoning.

Summary

Conditional Probability

A **conditional probability** is the probability that one event occurs under the condition that another event occurs.

For example, we will remove two marbles from a jar that contains 3 green marbles, 2 blue marbles, 1 white marble, and 1 black marble. We might consider the conditional probability that the second marble we remove is green given that the first marble removed was green. The notation for this probability is $P(\text{green second} \mid \text{green first})$ where the vertical line can be read as "under the condition that the next event occurs" or "given that the next event occurs." In this example, $P(\text{green second} \mid \text{green first}) = \frac{2}{6}$ since we assume the condition that the first marble drawn was green has happened, so the second draw only has 2 possible green marbles left to draw out of 6 marbles still in the jar.

To find the probability of two events happening together, we can use a multiplication rule:

$$P(\text{A and B}) = P(\text{A} \mid \text{B}) \cdot P(\text{B})$$

For example, to find the probability that we draw two green marbles from the jar, we could write out the entire sample space and find the probability from that or we could use this rule.

$$P(\text{green second and green first}) = P(\text{green second} \mid \text{green first}) \cdot P(\text{green first})$$

Since the probability of getting green on the first draw is $\frac{3}{7}$ and the conditional probability was considered previously, we can find the probability that both events occur using the multiplication rule.

$$P(\text{green second and green first}) = \frac{2}{6} \cdot \frac{3}{7}$$

This tells us that the probability of getting green marbles in both draws is $\frac{1}{7}$ $\left(\text{since this is equivalent to } \frac{6}{42}\right)$.

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In cases where events A and B are independent, $P(A \mid B) = P(A)$ since the probability does not change whether B occurs or not. In these cases, the multiplication rule becomes:

$P(A \text{ and } B) = P(A) \cdot P(B)$

For example, when flipping a coin and rolling a standard number cube, the events "getting a tails for the coin" and "getting 5 for the number cube" are independent. That means we can find the probability of both events occurring to be $\frac{1}{12}$ by using the multiplication rule.

$P(\text{heads and } 5) = \frac{1}{2} \cdot \frac{1}{6}$

Glossary

conditional probability

Practice

Conditional Probability

1. Jada rolls one standard number cube, then she rolls another standard number cube.

   a. What is the probability that she rolls a 5 on both number cubes?

   b. What is the probability that the second roll is a 5 under the condition that the first roll is a 6?

   c. What is the probability that the second roll is a 5 under the condition that the first roll is a 5?

   d. What is the probability that the second roll is not a 5?

   e. What is the probability that the first roll is a 5 and the second roll is not a 5?

2. There are four slices of pizza left to choose from. Each slice of pizza has one topping. Three of the slices have sausage as a topping and the fourth slice has pepperoni as a topping. Kiran randomly selects one slice then Mai randomly selects one slice. What is the probability that Mai selects a slice of pepperoni pizza under the condition that Kiran selects a slice of sausage pizza?

   (A) $\frac{1}{3}$

   (B) $\frac{1}{4}$

   (C) $\frac{2}{3}$

   (D) $\frac{2}{4}$

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3. Han's soccer team plays a soccer game in the morning. Lin's soccer team plays a soccer game in the afternoon against a different team than Han's soccer team played in the morning. Let A represent the event "Han's soccer team wins the morning game" and B represent the event "Lin's soccer team wins the afternoon game."

   a. Describe the meaning of $P(B \mid A)$.

   b. Do you think A and B are independent events? Explain your reasoning.

   c. If the events are independent, how are $P(B \mid A)$ and $P(B)$ related?

4. Each of the letters A through F are written on slips of paper and placed in a hat. Priya selects a slip of paper at random and then replaces it. Noah then selects a slip of paper at random. (Lesson 8-7)

   a. What is the probability that Priya selects a slip of paper labeled A?

   b. What is the probability that Noah selects a slip of paper labeled A?

   c. What is the probability that both Priya and Noah select a paper labeled A?

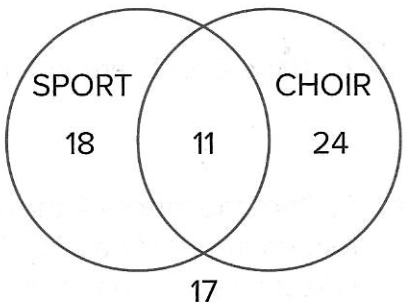
   d. Are the events of Priya selecting a paper and Noah selecting a paper dependent or independent? Explain your reasoning.

5. The Wildcats have won approximately 80% of their 20 basketball games this season. The Wildcats won 5 of the 8 games they played when Elena started the game. Are the events "the Wildcats win the game" and "the Wildcats win the game when Elena started the game" dependent or independent events? Explain your reasoning. (Lesson 8-7)

6. In a genetics experiment on plants, 17% of the plants exhibit trait A and 22% of the plants exhibit trait B. 36% of the plants exhibit trait A or trait B. What percentage of the plants exhibit both trait A and trait B? (Lesson 8-6)

7. 70 students were asked two survey questions:
   - Do you play a school sport?
   - Are you a member of the school choir?

   Their responses are summarized in the Venn diagram. (Lesson 8-5)



   a. How many students play a school sport?

   b. How many students play a school sport or are a member of the school choir?

   c. How many students play a school sport and are a member of the school choir?

   d. How many students are not a member of the school choir?

   e. Name two ways to determine how many students play a sport.

Lesson 8-9

Using Tables for Conditional Probability

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's use tables to estimate conditional probabilities.

Warm Up

9.1 Math Talk: Fractions in Fractions

Evaluate each expression mentally.

$\frac{7}{11} \div \frac{8}{11}$

$\frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)}$

$\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}$

$\frac{\left(\frac{3}{8}\right)\left(\frac{8}{19}\right)}{\left(\frac{5}{19}\right)}$

Activity

9.2 A Possible Cure

A pharmaceutical company is testing a new medicine for a disease using 115 test subjects. Some of the test subjects are given the new medicine and others are given a placebo. The results of their tests are summarized in the table.

	No More Symptoms	Symptoms Persist	Total
Given Medicine	31	26	57
Given Placebo	16	42	58
Total	47	68	115

1. Divide the value in each cell by the total number of test subjects to find each probability to two decimal places. Some of the values have been completed for you.

	No More Symptoms	Symptoms Persist	Total
Given Medicine	0.27		0.50
Given Placebo			
Total			1

If one of these test subjects is selected at random, find each probability:

   a. *P*(symptoms persist)

   b. *P*(given medicine and symptoms persist)

   c. *P*(given placebo or symptoms persist)

2. From the original table, divide each cell by the total for the row to find the probabilities with row conditions. Some of the values have been completed for you.

	No More Symptoms	Symptoms Persist	Total
Given Medicine	0.54		
Given Placebo			1

   a. *P*(symptoms persist | given medicine)

   b. *P*(no more symptoms | given placebo)

3. Jada didn't read the instructions for the previous problem well and used the table she created on the first problem to divide each cell by the probability total for each row. For example, in the top left cell she calculated $0.27 \div 0.5$. Complete the table using Jada's method.

	No more Symptoms	Symptoms Persist	Total
Given Medicine			
Given Placebo			

What do you notice about this table?

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4. From the original table, divide each cell by the total for the column to find the probabilities with column conditions. Some of the values have been completed for you.

	No More Symptoms	Symptoms Persist
Given Medicine	0.66	
Given Placebo		
Total		1

   a. *P*(given medicine | symptoms persist)

   b. *P*(given placebo | no more symptoms)

5. Are the events "symptoms persist" and "given medicine" independent events? Explain or show your reasoning.

6. Based on your work, does being given this medicine have an impact on whether symptoms persist or not?

Are you ready for more?

Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

1. 65% of the students who are going to prom are from grade 12. Complete the table.

	Going to Prom	Not Going to Prom	Total
Grade 11			127
Grade 12			116
Total	124	119	243

2. Based on your results, what is the probability that a student from this group, selected at random, is a student in grade 11 that did not go to prom?

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Activity

9.3 The Blood Bank

A blood bank in a region has some information about the blood types of people in its community. Blood types are grouped into type O, A, B, and AB. Each blood type either has the Rh factor (Rh+) or not (Rh−). If a person is randomly selected from the community, the probability of their having each blood type and Rh factor combination is shown in the table.

	O	A	B	AB
Rh+	0.374	0.357	0.085	0.034
Rh−	0.066	0.063	0.015	0.006

1. What does the 0.085 in the table represent?

2. Use the table or create additional tables to find the probabilities, then describe the meaning of the event.
   a. $P(\text{O})$
   b. $P(\text{Rh+})$
   c. $P(\text{O and Rh+})$
   d. $P(\text{O or Rh+})$
   e. $P(\text{O} \mid \text{Rh+})$
   f. $P(\text{Rh+} \mid \text{O})$

Summary

Using Tables for Conditional Probability

Organizing data in tables is a useful way to see information and compute probabilities. Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

	Going to Prom	Not Going to Prom	Total
Grade 11	43	84	127
Grade 12	81	35	116
Total	124	119	243

A student is randomly selected from this group of students. We can find a number of probabilities based on the table.

$P(\text{grade 12 and going to prom}) = \frac{81}{243}$ since there are 81 students who are both in 12th grade and going to prom out of all 243 students in the group who could be selected.

$P(\text{grade 12}) = \frac{116}{243}$ since there are 116 grade 12 students out of the entire group.

$P(\text{going to prom}) = \frac{124}{243}$ since there are 124 students going to prom out of the entire group.

$P(\text{grade 12 or going to prom}) = \frac{159}{243}$ since there are 159 students in grade 12 or going to prom (43 from grade 11 going to prom, 81 from grade 12 going to prom, and 35 from grade 12 not going to prom).

$P(\text{going to prom} \mid \text{grade 12}) = \frac{81}{116}$ represents the probability that the chosen student is going to prom under the condition that they are in grade 12. The group we are considering is different for this probability since the condition is that they are in grade 12. Imagine all grade 12 students are in a room and we are only selecting from this group. Therefore 81 students are going to prom from this group out of 116 students in the group.

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$P(\text{grade 11} \mid \text{going to prom}) = \frac{43}{124}$ represents the probability that the chosen student is in grade 11 under the condition that they are going to prom. In this case, we imagine that everyone going to prom is gathered in a room and we are selecting one student from this group. The probability that this student is in grade 11 when just considering this group is $\frac{43}{124}$.

The last two probabilities can also be found using the multiplication rule. $P(\text{A and B}) = P(\text{A} \mid \text{B}) \cdot P(\text{B})$ can be rewritten $P(\text{A} \mid \text{B}) = \frac{P(\text{A and B})}{P(\text{B})}$.

For example, substituting values into $P(\text{going to prom} \mid \text{grade 12}) = \frac{P(\text{going to prom and grade 12})}{P(\text{grade 12})}$ gives $\frac{81}{116} = \frac{\left(\frac{81}{243}\right)}{\left(\frac{116}{243}\right)}$ which is a true equation.

Practice

Using Tables for Conditional Probability

1. A tour company makes trips to see dolphins in the morning and in the afternoon. The two-way table summarizes whether or not customers saw dolphins on a total of 40 different trips.

	Morning	Afternoon
Dolphins	19	14
No Dolphins	3	4

   a. If a trip is selected at random, what is the probability that customers did not see dolphins on that trip?

   b. If a trip is selected at random, what is the probability that customers did not see dolphins under the condition that the trip was in the morning?

   c. Are the events of seeing dolphins and the time of the trip (morning or afternoon) dependent or independent events? Explain your reasoning.

2. Noah is unsure whether the coin and number cube he has are fair. He flips the coin then rolls the number cube and records the result. He does this a total of 50 times. The results are summarized in the table.

	One	Two	Three	Four	Five	Six
Heads	5	3	5	3	5	6
Tails	3	4	5	2	6	3

   a. Create a two-way table that displays probability for each outcome based on Noah's tests.

   b. If one of Noah's 50 results is selected at random, what is the probability that the coin was heads?

   c. If one of Noah's 50 results is selected at random, what is the probability that the number cube was 5?

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3. A student surveys 30 people as part of a project for a statistics class. Here are the survey questions.

   - Are you left-handed or right-handed?
   - Are you left-eye dominant or right-eye dominant?

   The results of the survey are summarized in the two-way table.

	Right-Eye Dominant	Left-Eye Dominant
Right-Handed	14	11
Left-Handed	3	2

   What is the probability that a person from the survey chosen at random is right-handed under the condition that they are right-eye dominant?

   (A.) $\frac{25}{30}$ (C.) $\frac{14}{25}$

   (B.) $\frac{14}{30}$ (D.) $\frac{14}{17}$

4. Priya flips a fair coin and then rolls a standard number cube. What is the probability that she rolled a 3 under the condition that she flipped heads? (Lesson 8-8)

   (A.) $\frac{1}{2}$ (C.) $\frac{1}{12}$

   (B.) $\frac{1}{6}$ (D.) $\frac{3}{12}$

5. Andre flips one fair coin and then flips another fair coin. (Lesson 8-8)

   a. What is the probability that he gets heads on both coins?

   b. What is the probability that he gets heads on the second coin under the condition that the first flip is heads?

   c. What is the probability that the second flip is not heads?

   d. What is the probability that the first flip is heads and the second flip is not heads?

6. Han randomly selects a card from a standard deck of cards. He places it on his desk and then Jada randomly selects a card from the remaining cards in the same deck. (Lesson 8-7)

   a. What is the probability that Han selects a card that has diamonds on it?

   b. What is the probability that Jada selects a card that has diamonds on it?

   c. What is the probability that Han selects a card that has diamonds on it and that Jada selects a card that has diamonds on it?

   d. Are the events of Han and Jada randomly selecting a card dependent or independent? Explain your reasoning.

7. An agriculturist takes 50 samples of soil and measures the levels of two nutrients, nitrogen and phosphorus. In 46% of the samples the nitrogen levels are low and in 28% of the samples the phosphorus levels are low. In 10% of the samples both the nitrogen and the phosphorus levels are low. What percentage of the samples have nitrogen levels or phosphorus levels that are low? (Lesson 8-6)

8. Select **all** of the situations that have a 50% chance of occurring. (Lesson 8-1)

   (A.) Rolling a standard number cube and getting a 3.

   (B.) Flipping two fair coins and getting heads on exactly one of the flips.

   (C.) Picking a letter at random from the word SEED and getting an E.

   (D.) Picking a letter at random from the word ORCHID and getting a vowel.

   (E.) Getting the answer correct when guessing randomly on a true or false question.

9. A solid has volume 6 cubic units and surface area 14 square units. The solid is dilated, and the image has surface area 224 square units. What is the volume of the image? (Lesson 5-8)

Lesson 8-10

Using Probability to Determine Whether Events Are Independent

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's take a closer look at dependent and independent events.

Warm Up

10.1 Which One Doesn't Belong: Events

A coin is flipped and a standard number cube is rolled. Which one doesn't belong?

Set 1

Event A1: the coin landing heads up

Event B1: rolling a 3 or 5

Set 2

Event A2: rolling a 3 or 5

Event B2: rolling an odd number

Set 3

Event A3: rolling a prime number

Event B3: rolling an even number

Set 4

Event A4: the coin landing heads up

Event B4: the coin landing tails up

Activity

10.2 Overtime Wins

Does a hockey team perform differently in games that go into overtime (or shootout) compared to games that don't? The table shows data about the team over 5 years.

Year	Games Played	Total Wins	Overtime or Shootout Games Played	Wins in Overtime or Shootout Games
2018	82	46	19	6
2017	82	46	18	7
2016	82	51	23	16
2015	82	54	18	10
2014	82	34	17	5
total	410	231	95	44

Let A represent the event "the hockey team wins a game" and B represent "the game goes to overtime or shootout."

1. Use the data to estimate the probabilities. Explain or show your reasoning.
   a. $P(A)$
   b. $P(B)$
   c. $P(A \text{ and } B)$
   d. $P(A \mid B)$
2. We have seen two ways to check for independence using probability. Use your estimates to check whether each might be true.
   a. $P(A \mid B) = P(A)$
   b. $P(A \text{ and } B) = P(A) \cdot P(B)$
3. Based on these results, do you think the events are independent?

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Activity

10.3 Genetic Testing

A suspected cause of a disease is a variation in a certain gene. A study gathers at-risk people at random and tests them for the disease as well as for the genetic variation.

	Has the Disease	Does Not Have the Disease
Has the Genetic Variation	80	12
Does Not Have the Genetic Variation	1,055	1,160

A person from the study is selected at random. Let A represent the event "has the disease" and B represent "has the genetic variation."

1. Use the table to find the probabilities. Show your reasoning.

   a. $P(A)$

   b. $P(B)$

   c. $P(A \text{ and } B)$

   d. $P(A \mid B)$

2. Based on these probabilities, are the events independent? Explain your reasoning.

3. A company that tests for this genetic variation has determined that someone has the variation and wants to inform the person that they may be at risk of developing this disease when they get older. Based on this study, what percentage chance of getting the disease should the company report as an estimate to the person? Explain your reasoning.

Are you ready for more?

Find or collect your own data that can be represented using a two-way table. Some ideas include finding data about a sports team (shooting percentage greater than 50% or less than 50% vs. winning or losing) or clinical trials for a medication (placebo or medication vs. symptom or no symptom).

1. Create a two-way table to represent the sample space.

2. Use probabilities to determine whether events summarized in the table are dependent or independent events.

3. When data is collected in an experiment or a survey it is often difficult to show evidence that events are dependent. Why do you think this occurs? Explain your reasoning.

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Summary

Using Probability to Determine Whether Events Are Independent

Although it may not always be easy to determine whether events are dependent or independent based on their descriptions alone, there are several ways to check for independence using probabilities.

One way to recognize independence is by understanding the experiment well enough to see if it fits the definition:

- Events A and B are independent if the probability of Event A occurring does not change whether Event B occurs or not.

An example of independence that can be found this way might be the events "a coin landing heads up" and "rolling a 4 on a number cube" when flipping a coin and rolling a standard number cube. Whether the coin lands heads up or not does not change the probability of rolling a 4 on a number cube.

A second way to recognize independence is to use conditional probability:

- Events A and B are independent if $P(A \mid B) = P(A)$

An example of independence that can be found this way might be the events "gets a hit on the second time to bat in a game" and "struck out in the first at bat in a game" for a baseball player in games for a season. By looking at what happens when the player has his second at bat, we can estimate that P(hit on second at bat) = 0.324 and by looking only at the second at bat after a strikeout, we can estimate that P(hit on second at bat | strike out on first at bat) = 0.324 so the events are independent.

Another way to recognize independence is to look at the probability of both events happening:

- Events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$

An example of independence that can be found this way might be the events "making the first free throw shot" and "making the second free throw shot" for a basketball player shooting two free throws after a foul. By looking at the outcomes of the two shots for the player throughout the year, we can estimate that P(make the first shot) = 0.72, P(make the second shot) = 0.72 and P(make the first shot and make the second shot) = 0.52. The events are independent since $0.72 \cdot 0.72 \approx 0.52$.

Practice

Using Probability to Determine Whether Events Are Independent

1. The two-way table shows the number of games played by a team divided into categories based on whether the team warmed up or not as well as whether they made more shots than they missed during the game.

	More Shots Made	More Shots Missed
Warmed Up	14	6
Didn't Warm Up	17	15

One of these games is selected at random. For each question, Event A is "game played when warmed up" and Event B is "more shots made than missed." Use the data to estimate the probabilities.

a. $P(A)$

b. $P(B)$

c. $P(A \text{ and } B)$

d. $P(A \mid B)$

e. Use $P(A \mid B) = P(A)$ and $P(A \text{ and } B) = P(A) \cdot P(B)$ to determine if the two events are dependent or independent. Show or explain your reasoning.

2. The two-way table shows the number of counties in northwestern Iowa where the average corn yield was more than 200 bushels per acre for the years 2016 and 2017.

	2016	2017
More Than 200	9	6
200 or Less	3	6

a. What does the value 3 represent in the table?

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b. What is the probability that a county in northwestern Iowa selected at random had an average corn yield of more than 200 bushels per acre under the condition that it is 2016?

c. For counties in northwestern Iowa, is having an average yield of more than 200 bushels per acre and the year being 2016 or 2017 dependent or independent events? Explain your reasoning.

3. Tyler finds a news article that says, "The price of gasoline has increased to more than $3.00 per gallon and the pay for truck drivers is less than it was last year at this time." Are the events "gasoline costing more than $3.00 per gallon" and "truck driver pay" dependent or independent events? Explain your reasoning.

4. The two-way table summarizes whether or not a cross country team had practice when it was raining and when it was not raining at the end of the school day. (Lesson 8-9)

	Cross Country Practice	No Cross Country Practice
Raining	6	2
Not Raining	12	1

a. When it was raining at the end of the school day, what is the probability that cross country practice was held?

b. When it was not raining at the end of the school day, what is the probability that cross country practice was held?

c. Are the events of "holding cross country practice" and "raining at the end of the school day" dependent or independent events? Explain your reasoning.

5. Kiran flips a fair coin. If it lands on heads, Kiran will toss the coin again. If it lands on tails, Kiran will roll a standard number cube. What is the probability that Kiran gets heads on his second toss under the condition that he got heads on his first toss? (Lesson 8-8)

(A.) 0

(B.) 0.25

(C.) 0.5

(D.) 1

6. Diego randomly selects a card from a standard deck of cards. He places it on his desk and then Clare randomly selects a card from the remaining cards in the same deck. (Lesson 8-7)

a. What is the probability that Diego selects a card that has hearts on it?

b. What is the probability that Clare selects a card that has hearts on it?

c. What is the probability that Diego selects a card that has hearts on it and that Clare selects a card that has hearts on it?

d. Are the events of Diego and Clare randomly selecting a card dependent or independent? Explain your reasoning.

7. A spinner is divided into 8 equal sections. 2 of them are bronze, 1 of them is silver, and 5 of them are gold. (Lesson 8-2)

a. What is the probability that it lands on bronze?

b. What is the probability that it lands on silver or gold?

c. What is the probability that it lands on platinum?

Lesson 8-11

Probabilities in Games

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's use probability with games.

Warm Up

11.1 Rock, Paper, Scissors

There is a classic game called "Rock, Paper, Scissors." Two people play by counting to 3 together, then making a hand gesture to resemble paper (hand flat, palm down), rock (fist), or scissors (two fingers extended).

- When paper and rock are shown, paper wins the round.
- When paper and scissors are shown, scissors wins the round.
- When rock and scissors are shown, rock wins the round.
- When both players show the same thing, the round is a tie.

Find a partner and play the game with them 10 times in a row. Record the number of times you have played the game, the name of your opponent, what each person shows in each round, and who is the winner. Find another partner and play another 10 times in a row.

Is the event "win the round" dependent on another event? Explain your reasoning.

Choose an event that you think might influence the probability of winning, then analyze the data using probability to determine whether the event you chose to study is independent of winning. Provide evidence to support your claim.

Activity

11.2 Guess Which Card

Your teacher will give you 3 index cards.

- Leave one card blank.
- Use a ruler to draw lines connecting opposite corners to make an X on one side of the second card.
- Use a ruler to draw lines connecting opposite corners to make an X on both sides of the third card.
- Put all three cards in the bag.

One partner will remove a card from the bag and place it on the desk immediately so that only one side of the card can be seen. The goal is to guess correctly which card is on the desk: the blank card, the card with an X only on one side, or the card with an X on both sides.

1. Noah is playing the game and is looking at a card that shows an X. He says, "I have a fifty-fifty chance of correctly guessing which card it is." Do you agree with Noah? Explain your reasoning.

2. Play the game many times with your partner, taking turns for who takes a card out of the bag. Record whether the side that shows has an X or is blank, which card you guess, and which card it actually is when you check. Continue to play until your teacher tells you to move on.

3. Use the results from your games to estimate
   P(the card had an X on both sides | the side showing had an X on it).
   Explain or show your reasoning.

4. Since all of the outcomes are known, find the actual probability
   P(the card had an X on both sides | the side showing had an X on it).
   Explain or show your reasoning.

NAME _____ DATE _____ PERIOD _____

Practice

Probabilities in Games

1. The two-way table shows the number of eggs laid by 10 Rhode Island red chickens and 10 leghorn chickens in July and August.

	Eggs Laid in July	Eggs Laid in August
Rhode Island Red Chicken	247	252
Leghorn Chicken	239	241

A chicken is selected at random. For each question, Event A is "egg laid by a Rhode Island red chicken" and Event B is "egg laid in July." Use the data to estimate the probabilities. (Lesson 8-10)

a. $P(A)$

b. $P(B)$

c. $P(A \text{ and } B)$

d. $P(A \mid B)$

e. Use $P(A \mid B) = P(A)$ and $P(A \text{ and } B) = P(A) \cdot P(B)$ to determine if the two events are dependent or independent. Show or explain your reasoning.

2. Clare finds an article in the online student newspaper that says, "There are more students in the 9th grade this year than last year and there are more students in the 12th grade who are graduating this year than last year." Are the events "the number of students in the 9th grade" and "the number of 12th graders who are graduating" dependent or independent events? Explain your reasoning. (Lesson 8-10)

3. The two-way table summarizes whether or not a softball team had practice when it was raining and when it was not raining at the start of the day.
(Lesson 8-9)

	Softball Practice	No Softball Practice
Raining	4	1
Not Raining	12	3

a. When it was raining at the start of the day, what is the probability that softball practice was held?

b. When it was not raining at the start of the day, what is the probability that softball practice was held?

c. Are the events of "holding softball practice" and "raining at the start of the day" dependent or independent events? Explain your reasoning.

4. Mai rolls a standard number cube and then flips a fair coin. What is the probability that Mai flips heads under the condition that she rolls a 5?
(Lesson 8-8)

(A.) $\frac{1}{2}$

(B.) $\frac{5}{6}$

(C.) $\frac{1}{12}$

(D.) $\frac{5}{12}$

NAME _____ DATE _____ PERIOD _____

5. A total of 40 elementary, middle, and high school students participate in a fun run as a fundraiser. They are surveyed after the fun run to find out how many of them completed the fun run without walking. The results of the survey are shown in the table.

	Elementary	Middle	High
Walked During the Fun Run	8	2	1
Did Not Walk During the Fun Run	4	14	11

Mai, wants to know the probability that one of the participants in the fun run selected at random is an elementary school student or did not walk during the fun run. To figure this out, she adds the three values in the second row of the table (4, 14, and 11) to the two values listed under the heading "elementary school" (8 and 4). She then divides that answer by 40 and obtains a probability of $\frac{41}{40}$. Mai realizes that $\frac{41}{40}$ is greater than 1 and determines that she must have made a mistake. (Lesson 8-6)

a. What is Mai's mistake? Explain your reasoning.

b. How does the addition rule account for this kind of mistake?

6. Two classes of middle school students who are going on a field trip were asked if they wanted to go to a science museum or an art museum. Each student selects one museum option. The table summarizes the museum preference of each student in the class.

	Science Museum	Art Museum
Class A	14	12
Class B	11	14

What is the probability that a student in class B selected at random prefers to go to the art museum? (Lesson 8-5)

7. Elena decides which type of pizza to order. The choices for crust are thin crust, stuffed crust, or regular crust. The choices for one topping are pepperoni, mushrooms, olives, sausage, meatballs, pineapple, or green peppers. Elena has trouble deciding because there are so many possibilities. She selects the type of crust and one topping at random. How many outcomes are in the sample space? (Lesson 8-3)

(A.) 3

(B.) 7

(C.) 10

(D.) 21

Learning Targets

Lesson	Learning Target(s)
8-1 Up to Chance	• I can find or estimate probability using a model or data from a chance experiment. • I can identify chance experiments.
8-2 Playing with Probability	• I can find the sample space for chance experiments. • I can model situations using probability. • I can use sample space to calculate probability.
8-3 Sample Spaces	• I can create organized lists, tables, and tree diagrams and use them to calculate probabilities.
8-4 Tables of Relative Frequencies	• I can use information in a two-way table to find relative frequencies and to estimate probability.
8-5 Combining Events	• I can use tables and Venn diagrams to represent sample spaces and to find probabilities.

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Lesson	Learning Target(s)
8-6 The Addition Rule	• I can use the addition rule to find probabilities.
8-7 Related Events	• I can estimate probabilities, including conditional probabilities, from two-way tables.
8-8 Conditional Probability	• I can estimate probabilities, including conditional probabilities, from two-way tables.
8-9 Using Tables for Conditional Probability	• I can estimate probabilities, including conditional probabilities, from two-way tables. • I can use probabilities and conditional probabilities to decide if events are independent.
8-10 Using Probability to Determine Whether Events Are Independent	• I can collect data and use it to estimate probabilities. • I can use probabilities to decide if events are independent.

Notes:

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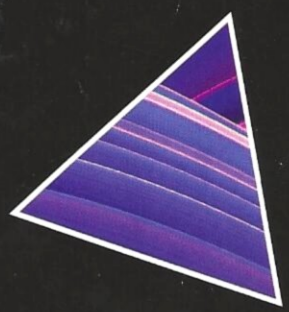

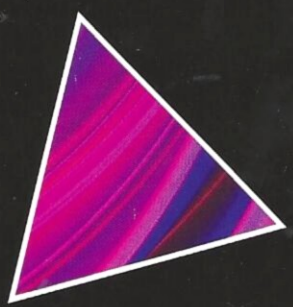

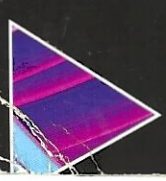

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